Project 2

Project 2: audio equalizer

Introduction

This project aims to motivate our study of filters by considering the design and implementation of an *audio equalizer*. An equalizer (EQ) modifies the frequency response of an audio system by amplifying or attenuating different frequency ranges. A simple EQ consists of bass and treble controls as is commonly found on stereo systems (Fig. 1). These are easily implemented with analog filters.

Fig. 1: Kinter MA-170 stereo amplifier with bass and treble controls.

Multiband EQs allow more precise tailoring of the frequency response and are often implemented using DSP (Fig. 2). These are also called *graphic EQs* because the positions of the slider bars provide a graphical representation of the frequency response. Graphic EQs can be used to compensate for room acoustics, microphone or speaker frequency resonances, or to tailor sound to a listener's preferences.

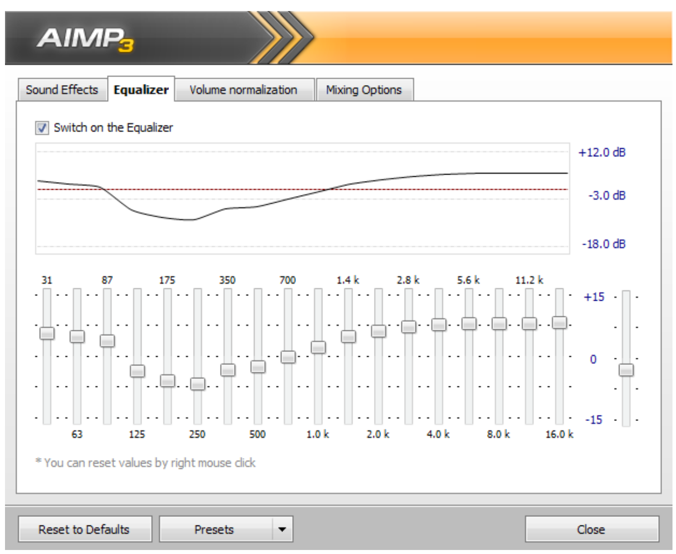

Fig. 2: DSP-based multiband equalizer. The bandpass filters are spaced at half-octave intervals, ($\sqrt{2}$ factors).

A *parametric EQ* allows independent adjustment of various parameters such as center frequency, bandwidth and gain.

Bass and treble

One approach to implementing bass and treble controls (a “two-band EQ”) is illustrated in Fig. 3. The input signal is simultaneously lowpass and highpass filtered, the filtered signals are amplified (or attenuated) as desired, and the results are added to produce the output signal.

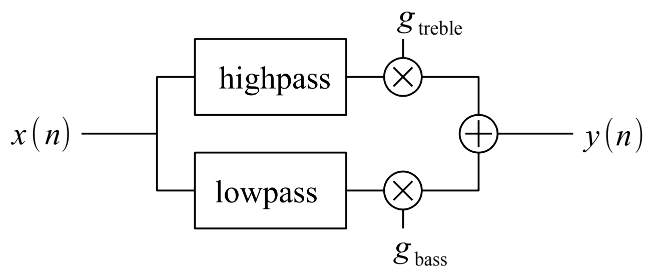

Fig. 3: Bass and treble control implemented with lowpass filter, highpass filter, bass gain and treble gain.

A literal description of the system is

$$y(n) = g_{\text{bass}} h_{LP}(n) * x(n) + g_{\text{treble}} h_{HP}(n) * x(n) \tag{1}$$

which requires two convolutions. However, convolution is a linear operation, and we can “factor out” the input signal

$$y(n) = [g_{\text{bass}} h_{LP}(n) + g_{\text{treble}} h_{HP}(n)] * x(n) \tag{2}$$

This is a single convolution of the input

$$y(n) = h(n) * x(n) \tag{3}$$

with the impulse response

$$h(n) = g_{\text{bass}} h_{LP}(n) + g_{\text{treble}} h_{HP}(n) \tag{4}$$

This idea extends to an EQ with any number of frequency bands. Therefore we can always implement an arbitrary EQ operation with a single convolution.

A variation on the EQ filter is the *crossover* filter (Fig. 4). In this case we keep the frequency bands separated and send each to its own amplifier and speaker. In a high-quality audio system this is necessary because it is very difficult to build a single speaker that can faithfully reproduce the entire audio spectrum. Instead, high-quality speakers are typically designed for a limited frequency range, and two or more speakers must be combined in a speaker cabinet to reproduce the entire audio signal.

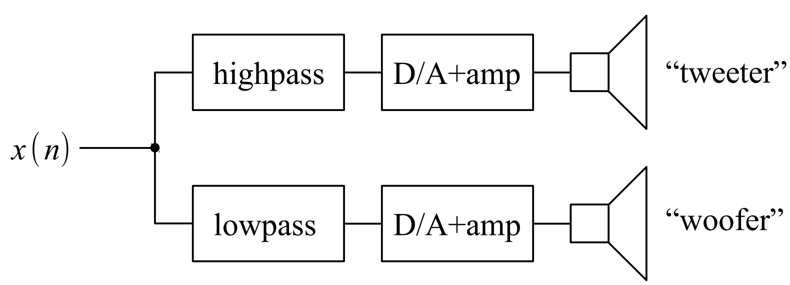

Fig. 4: Two-way crossover filter.

For a two-band EQ or crossover filter we, somewhat arbitrarily, will take

$$F_c = 1000\,\text{Hz} \tag{5}$$

to be the "crossover" or "cutoff" frequency. With $F_s = 44.1\,\text{kHz}$ the corresponding digital frequency is

$$f_c = \frac{1\,\text{kHz}}{44.1\,\text{kHz}} = 0.02268 \tag{6}$$

To implement the system of Fig. 3 or Fig. 4 we need to design lowpass and highpass filters with this cutoff frequency.

Lowpass filter

A lowpass filter should pass a low-frequency signal (signal that changes slowly) and block a high-frequency signal (signal that changes rapidly). Let's consider the input

$$x(n) = e^{j\omega n} = e^{j2\pi f n} \tag{7}$$

At the lowest frequency $f = \omega = 0$ we have

$$x(n) = 1 \tag{8}$$

while at the highest frequency $f = 1/2$, $\omega = \pi$ we have

$$x(n) = (-1)^n \tag{9}$$

A simple filter that passes (8) and blocks (9) is

$$y(n) = \frac{1}{2}[x(n) + x(n-1)] \tag{10}$$

The frequency response of this filter

$$H(f) = \frac{1}{2}(1 + e^{-j2\pi f}) = e^{-j\pi f}\cos(\pi f) \tag{11}$$

is illustrated in Fig. 5.

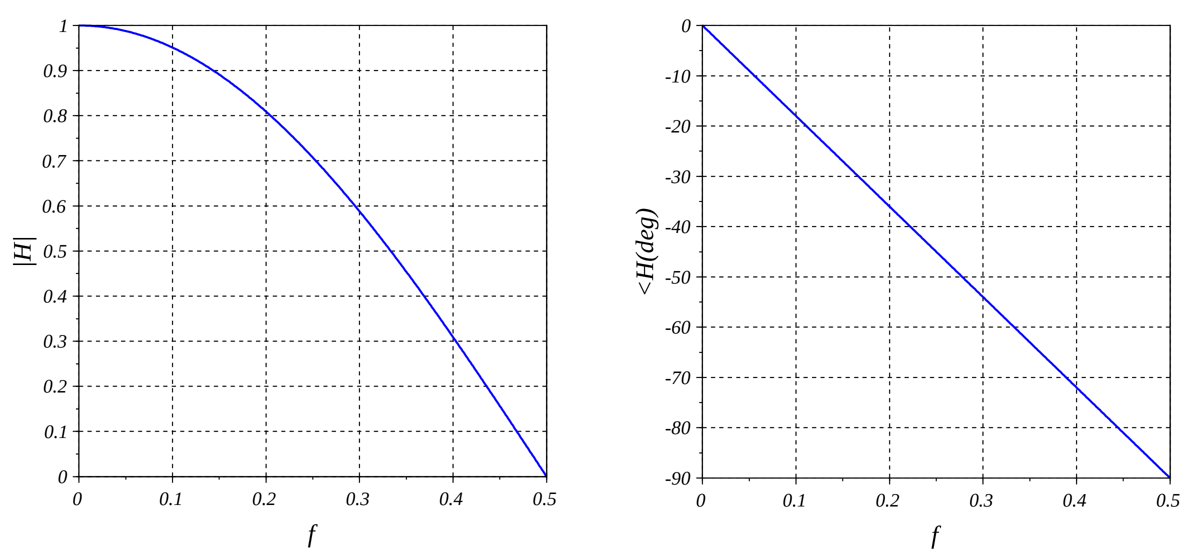

Fig. 5 Frequency response of filter (10).

Notice that the phase of the filter response is linear in f

$$\angle H(f) = -\pi f \tag{12}$$

Such a filter is said to be *linear phase*. This is often a desirable property. Suppose $|H(f)| = A(f)$ and $\angle H(f) = -2\pi a f$ is a linear-phase response. Then a sinusoidal input to the filter produces an output

$$\cos(2\pi f n) \rightarrow A(f)\cos(2\pi f n - 2\pi a f) = A(f)\cos(2\pi f[n-a]) \tag{13}$$

that is shifted in time by a samples, regardless of frequency.

The response of (10) is rather crude. We can achieve an improved response by adding more terms

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) \tag{14}$$

This is the FIR filter we have discussed previously. The problem then is to determine the coefficients b_k that result in the desired response. We will study this problem in detail. An example FIR filter response is shown in Fig. 6 at left.

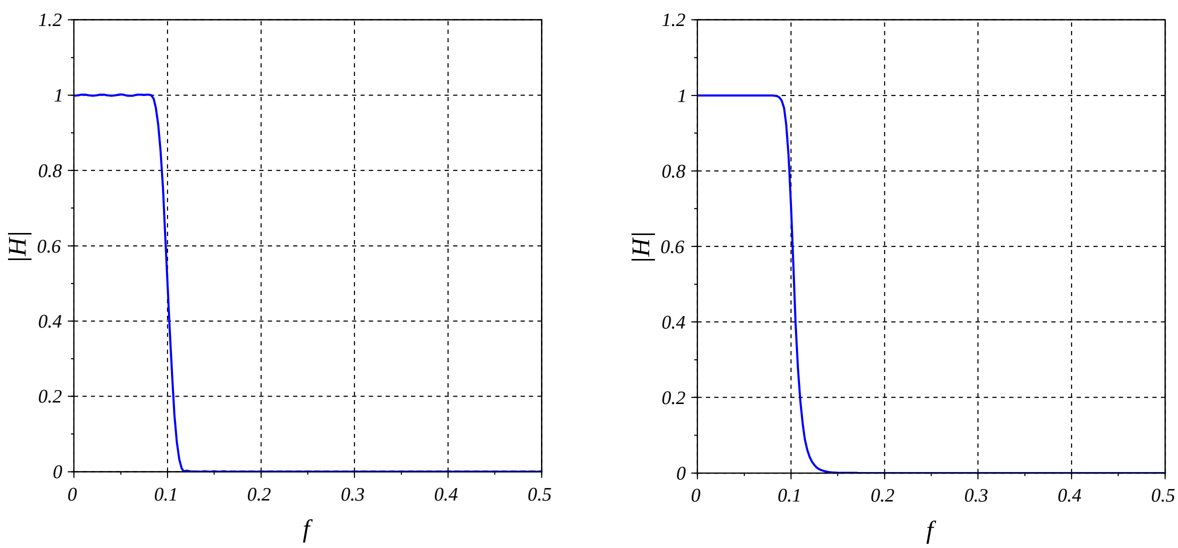

Fig. 6: Frequency response of (left) FIR lowpass filter with $M = 50$ *and* $f_c = 0.1$ *and (right) IIR lowpass filter with* $M = N = 12$.

We can also implement IIR filters

$$y(n) = \sum_{k=0}^{M} b_k x(n-k) - \sum_{k=1}^{N} a_k y(n-k) \tag{15}$$

Typically these allow us to obtain a similar frequency response with fewer filter coefficients (Fig. 6 at right).

Highpass filter

A highpass filter should block a low-frequency signal (signal that changes slowly) and pass a high-frequency signal (signal that changes rapidly). Changing (10) to

$$y(n)=\frac{1}{2}\left[x(n)-x(n-1)\right] \tag{16}$$

results in a frequency response

$$H(f)=\frac{1}{2}\left(1-e^{-j\pi f}\right)=j\,e^{-j\pi f}\sin(\pi f) \tag{17}$$

This is plotted below.

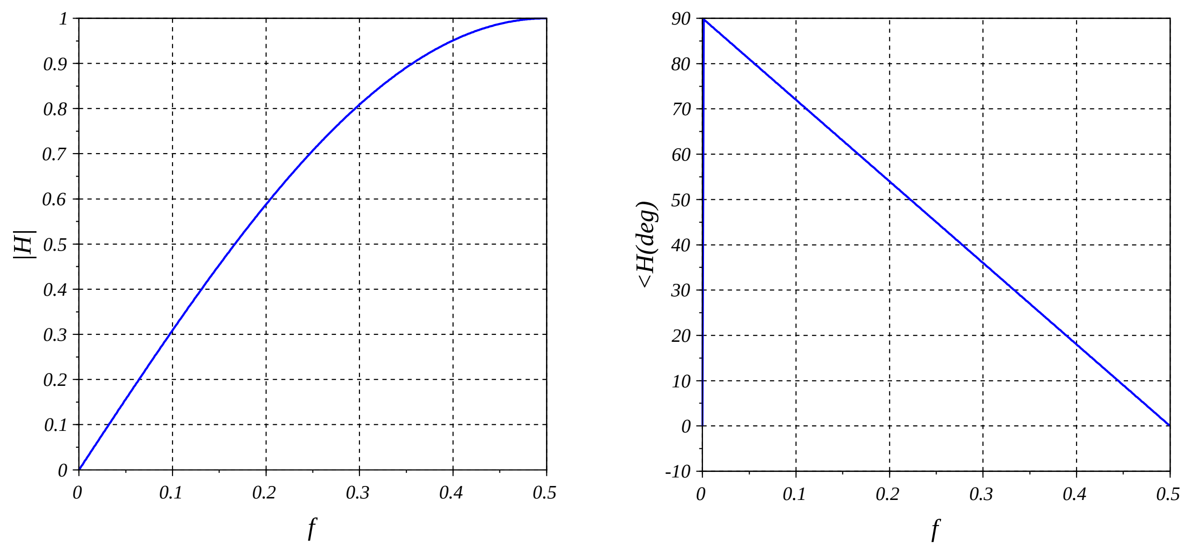

Fig. 7 Frequency response of filter (16).

As with the lowpass filter, we can add more filter coefficients to achieve a more desirable response, either with an FIR or IIR form.

N-band equalizer

An *N*-band equalizer is illustrated in Fig. 8. Here band 1 is isolated using a lowpass filter while band *N* is isolated using a highpass filter. All other bands require a *bandpass* filter. The resulting signal is

$$y(n)=g_1 h_1(n)*x(n)+g_2 h_1(n)*x(n)+\cdots+g_N h_N(n)*x(n)$$

This reduces to

$$y(n)=h(n)*x(n)$$

with

$$h(n)=g_1 h_1(n)+g_2 h_2(n)+\cdots g_N h_N(n)$$

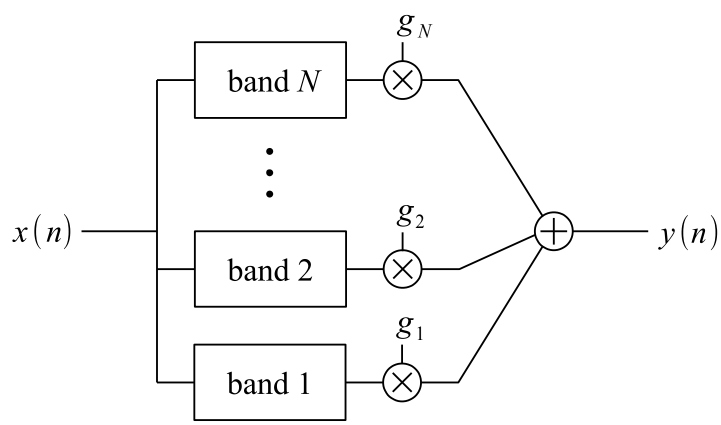

Fig. 8: N-band equalizer. Band 1 is a lowpass filter, Band N is a highpass filter, and the rest are bandpass filters. Each band has independent gain control.

As an example, the pass bands of an 8-band filter are shown in Fig. 9.

Bandpass filter

There are two approaches to designing a bandpass filter. One idea is to cascade a lowpass and a highpass filter with suitably chosen cutoff frequencies. This can be an attractive option when the passband is relatively large. For narrow-band filters it is usually advisable to design the entire filter as a single set of coefficients.

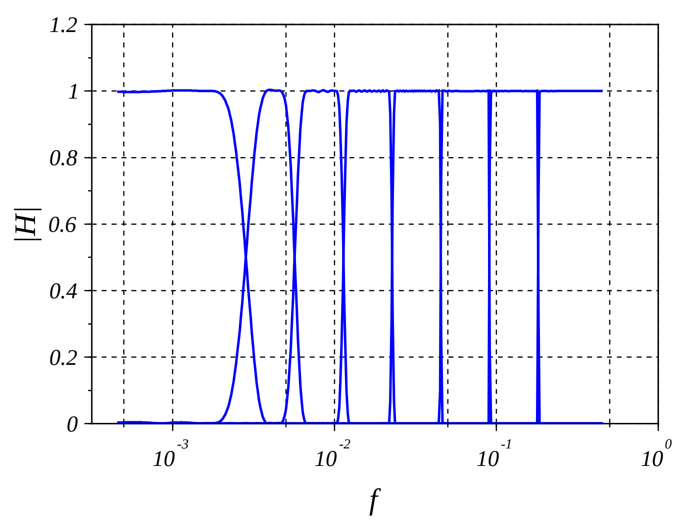

Fig. 9: Pass bands of an 8-band FIR equalizer ($M = 882$). Note logarithmic frequency scale. Band edges (in Hz) are: 125, 250, 500, 1k, 2k, 4k, 8k.