Direct Sequence Spread Spectrum II

BER

One might think that DS-SS would have the following drawback. Since the RF bandwidth is N_{bc} times that needed for a narrowband BPSK signal at the same data rate R_b, there will be N_{bc} times as much noise in the DS-SS receiver and the BER will go up accordingly. It is true that the wider spectrum of the DS-SS signal contains N_{bc} times as much noise, but just as the receiver processing allowed us to get rid of other DS-SS signal, so too it allows us to get rid of most of this noise. The signal plus noise is

$$s(t)+n(t) = A_c m(t)c(t)\cos\omega_c t + n(t) \tag{27.1}$$

After de-spreading we have

$$\left[s(t)+n(t)\right]c(t) = A_c m(t)\cos\omega_c t + n(t)c(t) \tag{27.2}$$

Consider the term $n(t)c(t)$. $n(t)$ is noise. Multiplication by $c(t)=\pm 1$ does not change the amplitude of $n(t)$, it simply causes some sign flips on an already random process. So we can just call this a new noise process $n_1(t)=n(t)c(t)$. $n_1(t)$ and $n(t)$ will have the same variance hence the same spectral density N_0. Then

$$\begin{aligned}\frac{2}{T_b}\int_{kT_b}^{(k+1)T_b}\left[A_c m(t)\cos\omega_c t + n_1(t)\right]\cos\omega_c t\,dt &= A_c m_k + \frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} n_1(t)\cos\omega_c t\,dt \\ &= A_c m_k + X\end{aligned} \tag{27.3}$$

Just as for narrowband BPSK, the last integral covers a time interval T_b hence it will include spectral components in a bandwidth $R_b = 1/T_b$ about the carrier frequency. Therefore $\sigma_n^{\ 2} = N_0/T_b$. It follows that the BER will be exactly the same as for BPSK

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{27.4}$$

We see that there is no BER penalty for using DS-SS. The fact the de-spreading processing effectively reduces the noise power from $N_0 B_{ss} = N_0 N_{cb} B_m$ to $N_0 B_m$, i.e., by a factor of N_{cb} can be though of as a *processing gain* of

$$G_{proc} = N_{cb} \tag{27.5}$$

However, keep in mind that in this situation the gain is just counteracting the increase in noise caused by the spreading. The important point is that there is no loss of power efficiency in using DS-SS.

Channelization

Now one might think, "Okay, there is no power efficiency penalty for DS-SS, but what about bandwidth efficiency? You have to use N_{cb} times as much bandwidth as you'd need to do narrowband BPSK at the same bit rate. So one user is the taking up bandwidth that could support N_{cb} users."

However, it turns out that many different links can operate over this bandwidth simultaneously. Suppose you want to have two links that transmit data streams $m_1(t)$ and $m_2(t)$ respectively. For example, a base station is to operate downlinks to two mobiles. Give each link its own PN sequence, say, $c_1(t)$ and $c_2(t)$. Then the transmitted signal is the sum of the two DS-SS signals.

$$s(t) = A_c\left[m_1(t)c_1(t) + m_2(t)c_2(t)\right]\cos\omega_c t \tag{27.6}$$

These occupy the same bandwidth B_{ss}. At the receiver we "tune" to one of these by multiplying by the corresponding spreading sequence. To tune to link 1 we'd have

$$s(t)c_1(t) = A_c m_1(t)\cos\omega_c t + A_c m_2(t)c_1(t)c_2(t)\cos\omega_c t \tag{27.7}$$

followed by

$$\frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} s(t)c_1(t)\cos\omega_c t\,dt = A_c\left[m_{1,k} + \frac{m_{2,k}}{T_b}\int_{kT_b}^{(k+1)T_b} c_1(t)c_2(t)dt\right] \tag{27.8}$$

Let's say that $c_i(t)$ is periodic with period T_b, that is it repeats after N_{cb} chips. Then the last integral above vanishes if

$$c_i \cdot c_j = \frac{1}{T_b}\int_0^{N_{cb}T_c} c_i(t)c_j(t)dt = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{27.9}$$

In other words we need to find two mutually orthogonal binary vectors, or "codes," of length N_{cb}. This is easy. Let $c_1(t)$ be any sequence of chips. Then we only need to make sure that ½ the chips in $c_2(t)$ are the same as the corresponding chip in $c_1(t)$ while the other ½ are different. Then $c_1 \cdot c_2 = N_{cb}/2 - N_{cb}/2 = 0$. For example, if the chips in $c_1(t)$ are $-1,1,-1,1$ then $c_2(t)$ could have chips $-1,1,1,-1$.

If we could find three mutually orthogonal codes, then three links could operate independently. How many mutually orthogonal binary vectors ("codes") of length N_{cb} can you have? This is analogous to asking how many mutually orthogonal vectors you can have in an N_{cb} dimensional space. The answer is N_{cb}. So, DS-SS uses N_{cb} times more bandwidth per user than narrow-band methods, but we can fit N_{cb} users into the same spectrum. There is no loss in bandwidth efficiency. Note, however, that this idea only works if the all the codes are aligned. You cannot find N_{cb} codes that are mutually orthogonal for arbitrary relative shifts. We discuss this below.

An example of sets of orthogonal binary vectors is the *Walsh codes.* The following recursion formula provides Walsh codes for any length equal to a power of 2.

$$H_1 = [0]$$
$$H_{2N} = \begin{bmatrix} H_N & H_N \\ H_N & \overline{H}_N \end{bmatrix} \tag{27.10}$$

Here H_N is an $N \times N$ matrix and $\overline{H}_N$ its negation, that is, you flips the bits $0 \leftrightarrow 1$. For example, the codes of length 2 and 4 are

$$H_2 = \begin{bmatrix} 0 & 0 \\ 0 & 1 \end{bmatrix}$$
$$H_4 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix} \tag{27.11}$$

The IS-95 CDMA system uses Walsh codes of length 64 to implement "channelization" of this sort on the downlink (from the base station to the mobiles). You might notice that the first code will always be all zeros. In this case $c(t) = const$ and there would be no spreading. This is true. However, following Walsh coding, all signals are multiplied by a so-called pseudo-random "short code." This achieves the desired spreading for all channels. More about this when we discuss CDMA cellular systems.

Unsynchronized Channels

The Walsh codes are orthogonal only if they are synchronized. Consider the last two rows of H_4. Notice that if we shift the fourth row $(0,1,1,0)$ to the right one chip we get $(0,0,1,1)$, which is the third row. So, if on uplinks we tried to use different Walsh codes for different mobiles, we could get a signal such as

$$s(t) = A_c\left[m_1(t)c_1(t) + m_2(t-t_2)c_2(t-t_2)\right]\cos\omega_c t \tag{27.12}$$

at the base station due to the different time delays on the two different links. We assume that the two amplitudes are the same. In practice, CDMA systems usually employ handset power control to try and achieve this, i.e., that signals from all mobiles arrive at the base station with equal power levels.

As we've just noted, $c_1(t)$ and $c_2(t)$ may be orthogonal, but $c_1(t)$ and $c_2(t-t_2)$ will not be for an arbitrary t_2. So, let's forget trying to have perfect orthogonality. Instead let the i^{th} link use its own PN spreading sequence $c_i(t)$ having a period much longer than N_{cb} chips. We require only that different sequences be uncorrelated. In that case $c_i(t)c_i(t) = 1$, but $c_i(t)c_j(t-t_0)$ is equally likely to be 1 or –1, i.e., is completely random. Now de-spread (27.12) with $c_1(t)$ to get

$$A_c m_1(t)\cos\omega_c t + A_c m_2(t-t_2)c_2(t-t_2)c_1(t)\cos\omega_c t \tag{27.13}$$

The second term here is random, i.e., noise, because $c_1(t)$ and $c_2(t)$ are uncorrelated. It has a bandwidth B_{ss} (because the spreading sequences do) centered on the carrier frequency. And, since $|m(t-t_2)c_2(t-t_2)c_1(t)|=1$ it has the same power as the first term (the signal). So, it is analogous to (27.1) where here the signal of mobile 2 appears as noise to mobile 1.

Assume we have N_m mobiles in a cell each producing P_s signal power at a base station. For now, neglect noise at the receiver. Instead, consider that for any signal, the other N_m-1 signals are effectively noise. Each has a power spectral density P_s/B_{ss}. Therefore the total power spectral density of the interference "noise" is

$$I_0=(N_m-1)\frac{P_s}{B_{ss}} \tag{27.14}$$

Since the bit energy is P_sT_b we have

$$\begin{aligned}\frac{E_b}{I_0}&=\frac{P_sT_b}{(N_m-1)\frac{P_s}{B_{ss}}}\\&=\frac{B_{ss}T_b}{N_m-1}\\&=\frac{N_{cb}}{N_m-1}\end{aligned} \tag{27.15}$$

For BPSK (and QPSK)

$$P_e=Q\left(\sqrt{\frac{2E_b}{I_0}}\right) \tag{27.16}$$

For a given BER this will fix the value of E_b/I_0. For example, $E_b/I_0=5$, or 7 dB, gives $P_e=Q(\sqrt{10})=0.08\%$. Then we can solve for the number of users that can be supported at this BER

$$N_m=1+\frac{N_{cb}}{E_b/I_0} \tag{27.17}$$

Notice that this is a "soft" limit. Equation (27.15) shows that each time we add a new user the E_b/I_0 degrades for all users, but there is no "hard" limit as there is in a system in which you run out of channels.

Equation (27.17) would be valid for a single cell with no outside interference and no receiver noise. In CDMA systems, however, all cells use the same frequency. Each mobile uses a different code. It is not simple to calculate the interference from other cells. This can be accounted for by a factor β. If users in our cell generate interference I, and if all other cells have a similar load, then the interference to our cell from all other cells is βI . Based on simulations, a rough estimate is that β is on the order of 1. At the same time, a phone channel is only active for a fraction of the time. A well-designed system can effectively stop transmitting during silence. If α is the fraction of time a voice channel is "active," then the interference is reduced by this factor. So the total interference from all sources with transmission only during voice activity, is approximately $\alpha(1+\beta)(N_m-1)P_s/B_{ss}$. Adding in the receiver noise spectral density, we have

$$\frac{E_b}{I_0}=\frac{P_sT_b}{\alpha(1+\beta)(N_m-1)\frac{P_s}{B_{ss}}+N_0} \tag{27.18}$$

Solving for the number of mobiles we have

$$N_m=1+\frac{1}{\alpha(1+\beta)}\left(\frac{N_{cb}}{E_b/I_0}-\frac{N_0B_{ss}}{P_s}\right) \tag{27.19}$$

Using "typical" numbers of $\alpha\approx 0.5$, $\beta\approx 1$, $E_b/I_0\approx 5$ this becomes

$$\begin{aligned} N_m &= 1+\frac{N_{cb}}{5}-\frac{N_0B_{ss}}{P_s} \\ &= 1+N_{cb}\left(\frac{1}{5}-\frac{N_0B_m}{P_s}\right) \end{aligned} \tag{27.20}$$

As an example, take $B_m=19.2\,\text{kHz}$ and $N_{cb}=64$. Then $B_{ss}=1.2288\,\text{MHz}$. In the limit of no noise ($N_0\to 0$) we get $N_m=1+64/5\approx 14$. If we use 120° sectoring, then this is the number of users per sector. The number of users per cell is therefore 42. Compare this to an analog system with a channel bandwidth of 30 kHz, no sectoring, and an $N=7$ reuse pattern. In this case a bandwidth of $1.2288\,\text{MHz}$gives us 41 total channels or about 6 per cell. Therefore the CDMA system gives 7 times the capacity per bandwidth relative to the analog system.

References

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