Direct Sequence Spread Spectrum

Introduction

Frequency-hopped spread spectrum communication is in a sense a logical extension of narrowband communication – you just switching channels periodically. The other main spread spectrum technique, called direct sequence spread spectrum (DS-SS), is inherently broadband. This technique seems quite strange at first sight, but it has a number of advantages that have lead to its becoming an increasing important method for implementing high-capacity, high-speed digital cellular communications.

Spreading

Central to DS-SS is the idea of *spreading*. Recall that in BPSK, a modulating signal $m(t)$ is constructed from data bits so that if the k^{th} bit is, say, 0 then $m(t) = -1$ for $kT_b \le t < (k+1)T_b$, where T_b is the bit period. Multiplying by a carrier $\cos\omega_c t$ creates the RF signal. This is illustrated at the top of Fig. 26.1

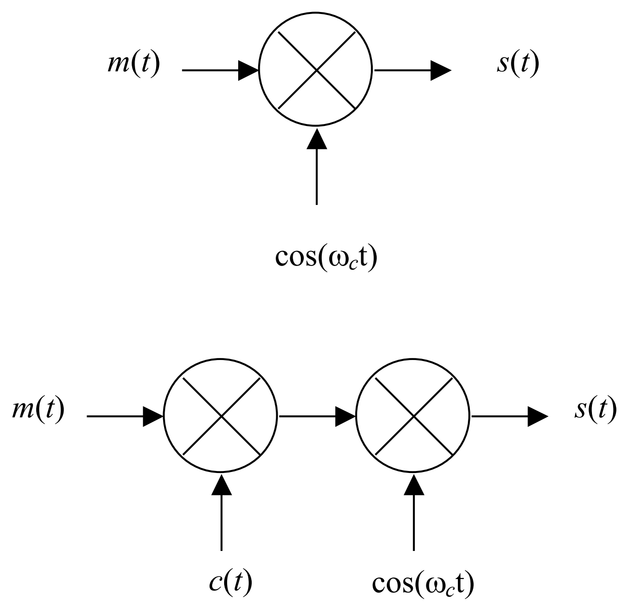

Figure 26.1: Top, regular BPSK. Bottom, multiplying the bit stream $m(t)$ by a spreading sequence spreads the signal over a much greater bandwidth.

To implement DS-SS modulation, we multiply $m(t)$ by another binary sequence, the *spreading sequence*, $c(t)$, followed by carrier modulation. This is illustrated at the bottom of Fig. 26.1. The binary values of $c(t)$ are called *chips*, and the chip period is $T_c = T_b / N_{cb}$ where N_{cb} is the number of chips per bit. The chips themselves do not convey information. The chip pattern is periodic and known to both the transmitter and receiver. The period of $c(t)$ may or may not be equal to the bit period.

Recall that the bit rate is $R_b = 1/T_b$ and the RF bandwidth required to transmit the BPSK signal is proportional to this. The *chip rate* is $R_c = 1/T_c = N_{cb}R_b$. The required RF bandwidth for the spread signal will be about N_{cb} times that required for the plain BPSK signal. This is similar to spreading factor M we had in FH-SS. For FH-SS, however, during a single hop the signal bandwidth was narrow. It was only when averaged over time that the bandwidth was uniformly spread. For DS-SS, however, the signal is inherently broadband because the sequence $c(t)$ is always varying rapidly.

At the receiver, the DS-SS signal can be "de-spread" by being multiplied by the known chip sequence. Since $c(t) = \pm 1$, $c^2(t) = 1$, and this multiplication removes all trace of the spreading. We end up with a simple BPSK signal that can be demodulated to recover the bit stream. This is illustrated in Fig. 26.2.

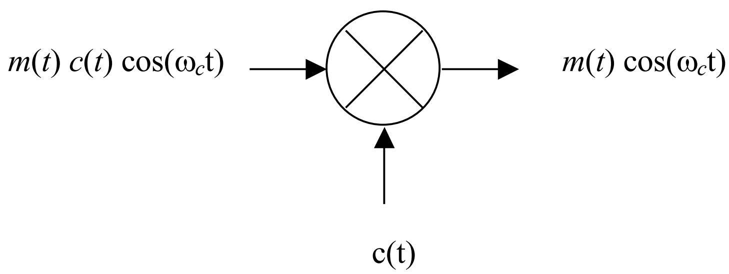

Figure 26.2: De-spreading at the receiver. This would most likely not be done at the actual RF carrier frequency, but at a lower "intermediate frequency." The effect is the same, however.

Example waveforms are shown in Fig. 26.3 for a case with $N_{cb} = 4$.

The spread signal varies more rapidly than the data by a factor of N_{cb}, i.e., $R_c = N_{cb}R_b$, so the bandwidth is correspondingly larger:

$$B_{ss} \approx N_{cb}B_m \tag{26.1}$$

where B_{ss} is the bandwidth of the spread spectrum signal and B_m is the bandwidth of the modulation. At the same time, the spectral intensity of the transmitted signal is decreased by a factor of $1/N_{cb}$. This is illustrated in Fig. 26.4.

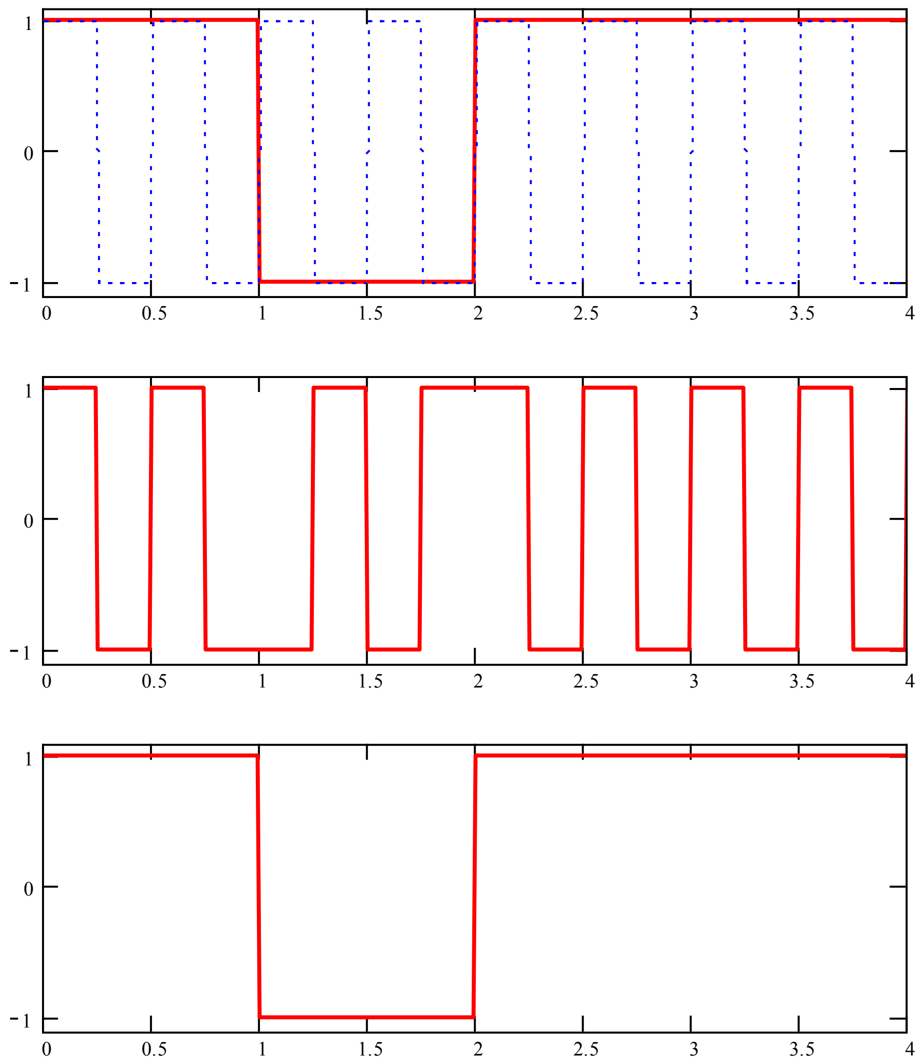

Figure 26.3: DS-SS example. At top, the thick (red) curve is the data stream $m(t)$ and the spreading sequence $c(t)$ is the dashed (blue) curve. In this case there are 4 chips per bit. The product, shown in the middle plot, forms the envelope of the spread spectrum signal. Multiplying again by the spreading sequence (at the receiver) recovers $m(t)$.

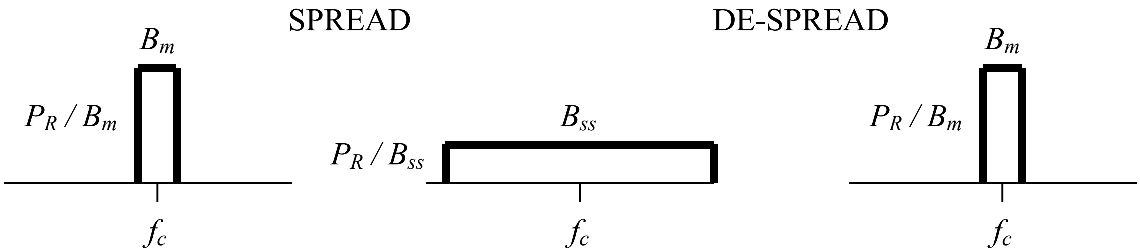

Figure 26.4: Spreading at the transmitter increases the bandwidth by a factor N_{cb} and decreases the spectral intensity by a factor $1/N_{cb}$ relative to a non-spread signal of the same data rate. De-spreading at the receiver re-concentrates the signal into the original bandwidth.

Mathematically, at the transmitter we create the DS-SS RF signal

$$s(t) = A_c m(t) c(t) \cos \omega_c t \tag{26.2}$$

At the receiver we multiply by the spreading sequence to obtain a BPSK signal

$$s(t)c(t) = A_c m(t) \cos \omega_c t \tag{26.3}$$

From this we can recover the data bit using

$$A_c m_k = \frac{2}{T_b} \int_{kT_b}^{(k+1)T_b} [m(t) \cos \omega_c t] \cos \omega_c t \, dt \tag{26.4}$$

So what is the point to all of this? Why not just skip the spreading/de-spreading steps and use the BPSK signal? For a cellular system, one of the biggest advantages is that DS-SS provides a means of combating fading due to multipath.

RAKE Filter

Recall that multipath results in two or more versions of the transmitted signal arriving at the receiver with different time delays. Consider a BPSK signal in the worst case where the two paths have the same amplitude, and differ by a time delay t_1 that gives them a phase difference of 180 degrees. We assume that $t_1 << T_b$ so that $m(t - t_1) \approx m(t)$. For example, for a bit rate of 19.2 kHz, T_b is 52 μs. A time delay this long would require a multipath difference of about 16 km. With this assumption the received signal is

$$\begin{aligned} r(t) &= A_c m(t) \cos \omega_c t - A_c m(t - t_1) \cos \omega_c (t) \\ &\approx A_c m(t) [\cos \omega_c t - \cos \omega_c t] \\ &= 0 \end{aligned} \tag{26.5}$$

and we suffer a severe fade and loss of signal. For the DS-SS system, our received signal is

$$\begin{aligned} r(t) &= A_c m(t) c(t) \cos\omega_c t - A_c m(t-t_1) c(t-t_1) \cos\omega_c(t) \\ &\approx A_c m(t)\left[c(t) - c(t-t_1)\right]\cos\omega_c t \end{aligned} \tag{26.6}$$

If $t_1 > T_c$ then the quantity in the brackets will not always be zero because it is the difference of two different chips. For example, if $N_{cb} = 64$ and T_b is 52 μs, then T_c is 0.81 μs corresponding to a multipath difference of only 240 m. This is illustrated in Figs. 26.5 and 26.6.

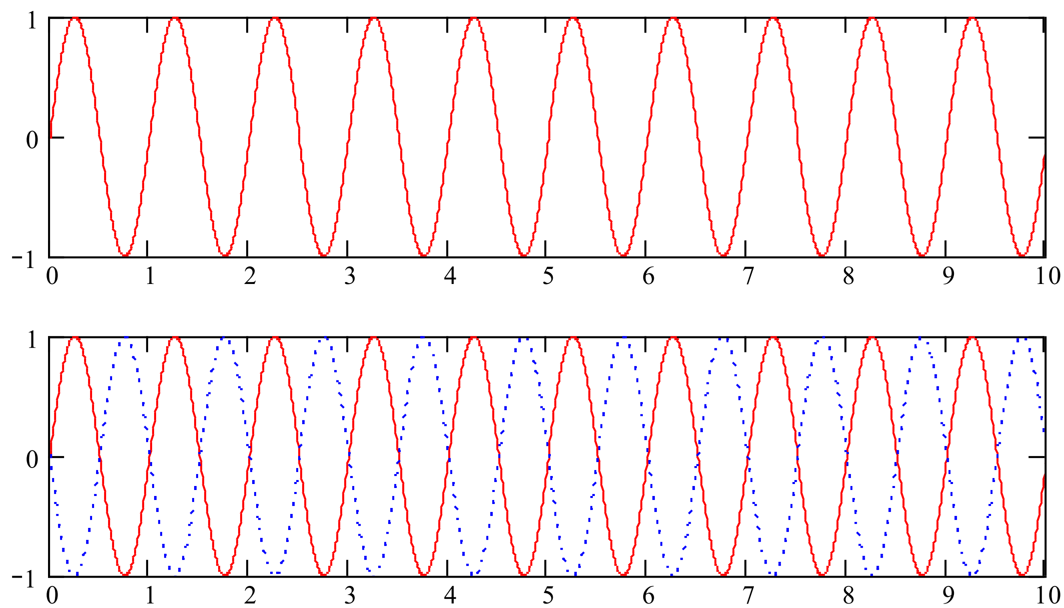

Figure 26.5: Multipath in a narrow band signal. One bit of the transmitted signal is at the top. The two received multipaths are at the bottom. The time delay puts them exactly out of phase. Their sum is zero at all times.

Since we have a non-zero received signal, we can try and recover the data. We "compute" the k^{th} data bit as

$$\begin{aligned} \frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} r(t)c(t)\cos\omega_c t\,dt &= \frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} A_c m(t) c^2(t)\cos^2\omega_c t\,dt - \frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} A_c m(t)c(t)c(t-t_1)\cos^2\omega_c t\,dt \\ &\approx A_c m_k\left\{1 - \frac{2}{T_b}\int_{kT_b}^{(k+1)T_b} c(t)c(t-t_1)\cos^2\omega_c t\,dt\right\} \end{aligned} \tag{26.7}$$

We'd like the last integral to go away. Using the fact that the average value of $\cos^2\omega_c t$ is ½, this becomes

$$\frac{1}{T_b}\int_{kT_b}^{(k+1)T_b} c(t)c(t-t_1)\,dt \approx 0 \tag{26.8}$$

This integral is the *autocorrelation* of the spreading sequence $c(t)$. It is the expected value of the product $c(t)c(t-t_1)$. Obviously if $t_1 = 0$ we have $\langle c(t)c(t-t_1)\rangle = 1$, but if $t_1 > T_c$ then $c(t)$ and $c(t-t_1)$ sample different chips of the spreading sequence. $\langle c(t)c(t-t_1)\rangle \approx 0$ says that we want the different chips (the sequence of ± 1 values in $c(t)$) to be uncorrelated. This means that they should appear "random." We call a sequence with this property a *pseudo-noise* (PN) sequence. There are simple digital techniques for generating such sequences.

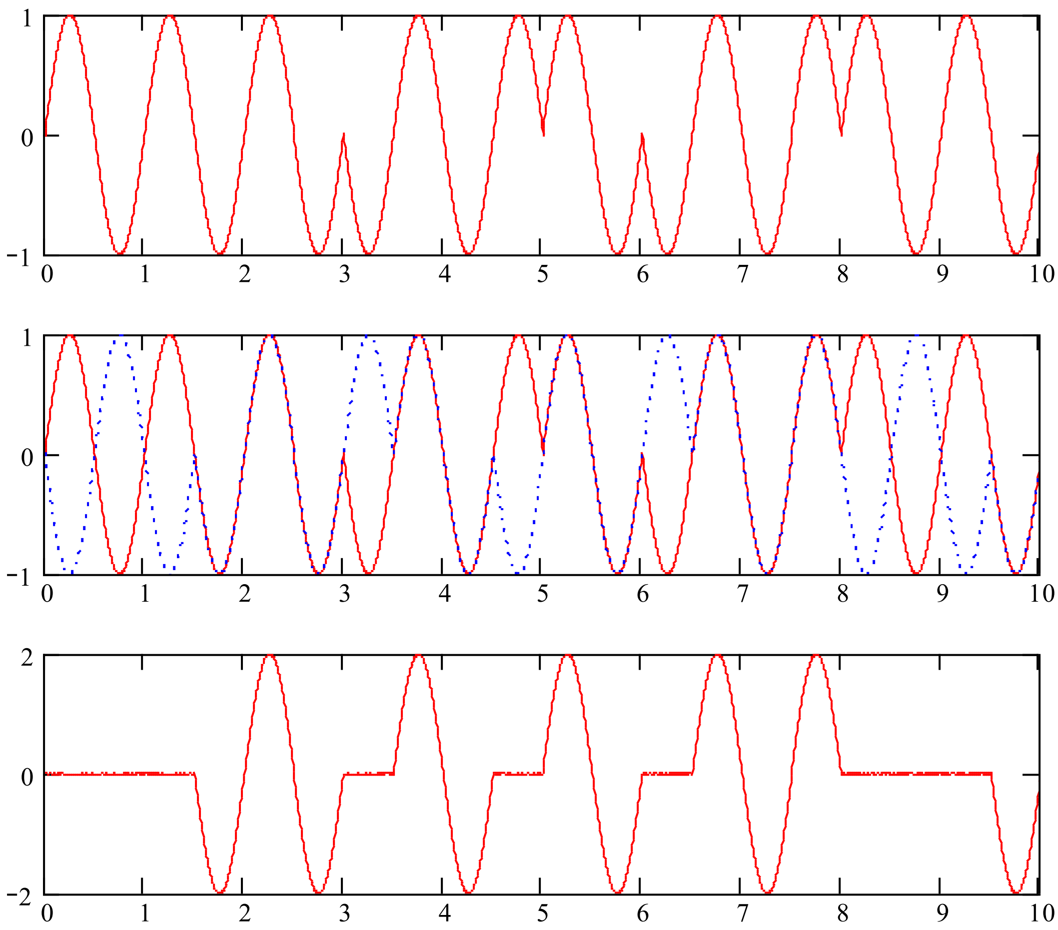

Figure 26.6: Multipath for a DS-SS signal. One bit of the transmitted signal is at the top. During the bit there are several phase flips due to the spreading sequence. The two received multipaths are in the center. The time delay was 1.5 chip periods. If this puts them out of phase at some times then they will be in phase at other times because $c(t)c(t-t_1)$ changes sign several times during the bit period. Therefore they do not completely cancel out as was the case for the narrowband signal.

Provided we use a noise-like sequence $c(t)$ for spreading we can recover the data bit m_k from the first multipath signal as if the second multipath wasn't even there! In fact, it's even better than that. We could also make the following "calculation"

$$-\frac{2}{T_b}\int_{t_1+kT_b}^{t_1+(k+1)T_b} r(t)c(t-t_1)\cos\omega_c t\,dt \approx A_c m_k\left\{-\frac{2}{T_b}\int_{t_1+kT_b}^{t_1+(k+1)T_b} c(t)c(t-t_1)\cos^2\omega_c t\,dt+1\right\} \tag{26.9}$$

In this case, provided again that $\langle c(t)c(t-t_1)\rangle \approx 0$, we recover the bit from the second path as if the first path wasn't there. So we get an independent estimate of the data bit. We could add these two results and get a stronger signal that we would have gotten in the absence of multipath! This idea is called a *RAKE receiver* and is illustrated in Fig. 26.7.

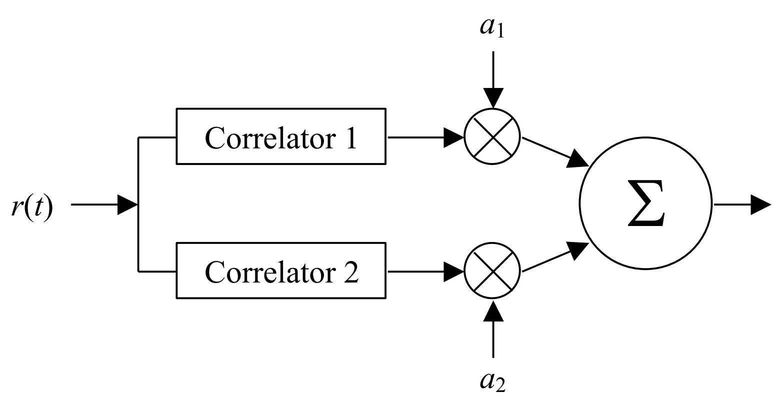

Figure 26.7: RAKE receiver. The received signal is processed by different correlators, each "tuned" to a different multipath time delay. Each correlator outputs an estimate of the data bit. These are optimally weighted and summed to get the best possible estimate. Three or more correlators can be used.

In this way DS-SS actually derives signal gain from multipath instead of suffering fading. Keep in mind, though, that this is only true for multipath delays greater than the chip period. For $t_1 << T_c$, $\langle c(t)c(t-t_1)\rangle \approx 1$ and multipath will cause fading. It follows that you'd like to make T_c as small as possible. This means using the highest possible chip rate, hence the largest possible bandwidth B_{ss}.

References

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