## QAM

## Introduction

QAM stands for Quadrature Amplitude Modulation. The idea is to use both phase and amplitude modulation to try and get more information through a channel of a given bandwidth. As we've noted before, trying to use $A M$ on a wireless channel is quite problematic due to the amplitude fluctuations introduced by fading. However, for point-to-point, or fixed links, this is often not a problem and QAM is feasible. QAM is also used extensively for modems.

If

$$
\begin{equation*}
s(t)=a\left\lfloor m_{I}(t) \cos \omega_{c} t-m_{Q}(t) \sin \omega_{c} t\right\rfloor \tag{24.1}
\end{equation*}
$$

the amplitude is $A(t)=a \sqrt{m_{I}^{2}(t)+m_{Q}^{2}(t)}$ and the phase is $\tan \phi(t)=m_{Q}(t) / m_{I}(t)$. Allowing $m_{I}, m_{Q}$ to take on values other than $\pm 1$ will in general result in both phase and amplitude variations. For example, let $m_{I}, m_{Q}$ take on any of the four values $-3,-1,1,3$. Then there are 16 possible combinations of $m_{I}, m_{Q}$. The result is $16-Q A M$. The constellation diagram is shown below. An example 16-QAM signal is shown in Fig. 24.2


Figure 24.1: 16-QAM constellation diagram. The bit labels are a Gray code designed so that a symbol error causes only one bit error.

Since there are sixteen constellation points, each point or symbol, can represent four bits. Demodulation is now a bit trickier. We would calculate amplitudes along both the $\cos \omega_{c} t$ and $-\sin \omega_{c} t$ axes of the constellation diagram. We then find which constellation point we are closest
to and assume that was the symbol transmitted. For example, the gray box in Fig. 24.1 would represent the "decision region" for the symbol 1000. We make an error if the noise is large enough to take us out of this box. If $a$ is the distance illustrated, then for the 1000 symbol, the probability of error would be $P_{e, s y m}=4 Q\left(a / \sigma_{n}\right)$. The factor of 4 comes from the fact that an error can be made by crossing any of the 4 sides of the square. For some of the symbols (e.g., 1101) you can only make an error by crossing two sides, so the factor of 4 would become 2 . If you average over all the symbols you find that, on average, you have a factor of 3 . Since things are arranged so that a symbol error results in only a single bit error (out of 4 total bits), the total BER for 16-QAM is

$$
\begin{equation*}
B E R=\frac{3}{4} Q\left(\sqrt{\frac{4 E_{b}}{5 N_{0}}}\right) \tag{24.2}
\end{equation*}
$$

The factor of $4 / 5$ in the radical comes from calculating the relationship between $a$ and the average symbol energy.


Figure 24.2: Example of 16-Qam signal.
If QAM operates with a symbol period of $T_{s}$, the spectrum is a sinc of null-to-null width $1 / T_{s}$. However, if there are $2^{M}$ points in the constellation diagram, then each symbol conveys $M$ bits. So the bandwidth efficiency is $\eta=M / 2$. For 16-QAM this is $\eta=2$, or twice that of QPSK. It is possible to use larger values of $M$. The drawback is that the BER degrades and the power efficiency decreases.

## References

1. Burr, A., Modulation and Coding for Wireless Communications, Prentice Hall, 2001, ISBN 0-201-39857-5.
