QPSK

The idea of QPSK is to superimpose two orthogonal BPSK signals within the same spectrum. Consider the two BPSK signals $m_I(t)\cos\omega_c t$ and $m_Q(t)\sin\omega_c t$. Although these have the same carrier frequency, they can be separated in a receiver because they are orthogonal. Assume that $m_I(t), m_Q(t)$ are constants over a period T_s (the *symbol period*). Then

$$\begin{aligned} \frac{2}{T_s}\int_0^{T_s}\left[m_I\cos\omega_c t + m_Q\sin\omega_c t\right]\cos\omega_c t\,dt &= m_I(t) \\ \frac{2}{T_s}\int_0^{T_s}\left[m_I\cos\omega_c t + m_Q\sin\omega_c t\right]\sin\omega_c t\,dt &= m_Q(t) \end{aligned} \tag{23.1}$$

So we can transmit two independent BPSK signals over the same spectrum. If we have a single bit stream $m(t)$ we could, for example, assign the first bit to $m_I(t)$, the second to $m_Q(t)$, the third to $m_I(t)$, and so on.

Rewriting (23.1) we have

$$\begin{aligned} s(t) &= \frac{A_c}{\sqrt{2}}\left[m_I(t)\cos\omega_c t - m_Q(t)\sin\omega_c t\right] \\ &= A_c\cos\left[\omega_c t + \phi(t)\right] \end{aligned} \tag{23.2}$$

where $\tan\phi(t) = m_Q(t)/m_I(t)$. Since $m_I, m_Q = \pm 1$, ϕ can take on one of the four values $k\pi/2 + \pi/4$, where $k = 0,1,2,3$. The corresponding constellation diagram is shown in Fig. 23.1. The corresponding time-domain signal is shown in Fig. 23.2. This is called *quadrature phase shift keying* or QPSK.

Each of the four possible phases corresponds to a *symbol* that represents two bits. The bit rate is therefore

$$R_b = \frac{2}{T_s} \tag{23.3}$$

The energy during a single symbol period is equally divided among the two bits, so

$$\begin{aligned} E_s &= \frac{A_c^2}{2}T_s \\ &= 2E_b \end{aligned} \tag{23.4}$$

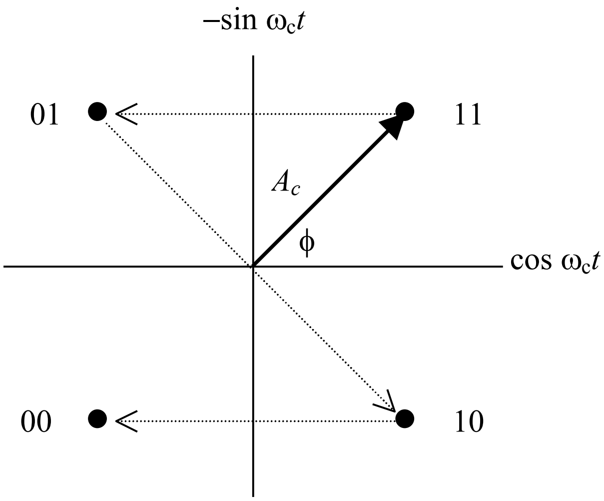

Figure 23.1: Constellation diagram for QPSK. Each phase represents a two-bit symbol. Transitions can occur in which one bit changes (horizontal dotted lines) or both bits change (diagonal dotted line).

QPSK doubles the bit rate of BPSK, but one price is that carrier recovery is no longer as simple. Square the signal we have

$$\begin{aligned}\left[m_I(t)\cos\omega_c t - m_Q(t)\sin\omega_c t\right]^2 &= \cos^2\omega_c t + \sin^2\omega_c t - 2m_I(t)m_Q(t)\cos\omega_c t\sin\omega_c t \\ &= 1 - m_I(t)m_Q(t)\sin(2\omega_c t)\end{aligned} \tag{23.5}$$

Unlike the BPSK case, this does not get rid of the modulation. More difficult techniques are required for carrier recovery with QPSK.

Another way to view QPSK is obtained if you imaging rotating the constellation diagram by $\pm 45°$. Then the constellation points would fall on the I and Q axes. We would be transmitting one of the four signals $\pm\cos(\omega_c t), \pm\sin(\omega_c t)$ to represent each two-bit symbol. Our choice of a phase reference is arbitrary, so there is nothing physical different about this scheme, it's just a different mathematical representation.

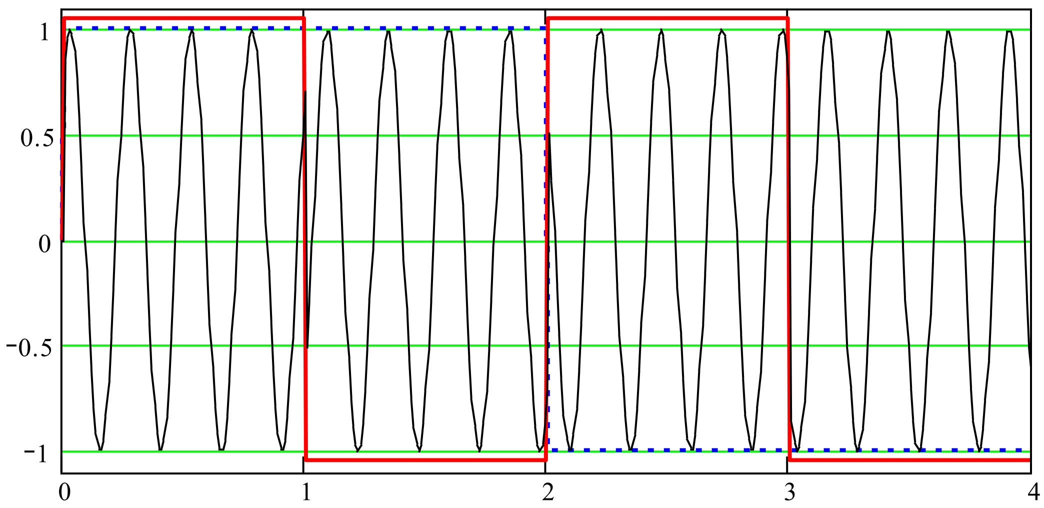

Figure 23.2: QPSK signal. The bit transitions correspond to the dotted path in Fig. 23.1.

Power and Spectral Efficiency

Since QPSK consists of two independent BPSK channels, the BER will be identical to the BPSK BER.

$$P_e = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{23.6}$$

So the power efficiency of BPSK and QPSK are identical. With regard to the spectrum, we have two orthogonal BPSK signals at the same carrier frequency. However, since two bits are sent per symbol, the symbol rate need only be ½ the bit rate. So, using our BPSK spectrum we have

$$\begin{aligned} |S(f)|^2 &= T_s \,\mathrm{sinc}^2\left[T_s(f-f_c)\right] \\ &= 2T_b \,\mathrm{sinc}^2\left[2T_b(f-f_c)\right] \end{aligned} \tag{23.7}$$

where $T_b = 1/R_b = T_s/2$ is the bit period that would be required for a BPSK signal at the same bit rate. Thus the QPSK spectrum is identical to the BPSK spectrum, but ½ the width. This is illustrated in Fig. 23.3 where it is compared to the MSK spectrum.

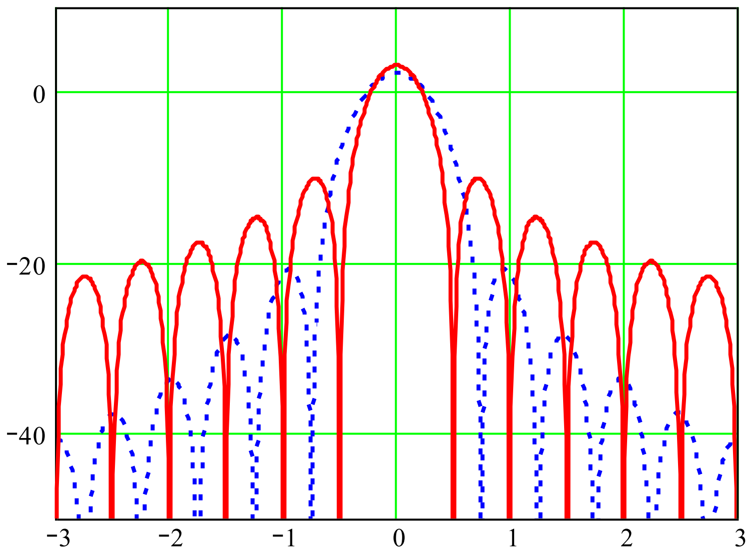

Figure 23.3: QPSK spectrum (solid red curve) compared to MSK spectrum (dashed blue curve) for same bit rate. Both curves have unit energy (on linear scale). Horizontal axis is normalized frequency fT_s relative to the carrier, vertical axis is amplitude in dB.

The spectral efficiency $\eta = R_b / B_{RF}$, with B_{RF} defined as the null-to-null width of the main lobe, is now 1. This is twice as efficient as BPSK and about 50% more efficient that MSK. But the price is relatively higher circuit complexity and more difficult carrier recovery.

An important application of QPSK is that it is used in the downlink (forward channel) of the IS-95 CDMA system in the US.

OQPSK

As always, the sinc spectrum has very large sidelobes. This is due to the sharp phase transitions in unfiltered phase shift keying. With respect to the constellation diagram (Fig. 23.1) this means that we move instantaneously from one constellation point to the next. To reduce the sidelobes we can filter $m_I(t), m_Q(t)$ resulting in continuous, smooth transitions between phases. On the constellation diagram of Fig. 23.1 this would mean moving along the dotted lines in a continuous fashion. This is illustrated in Fig. 23.4. If both $m_I(t)$ and $m_Q(t)$ transition at the same time, the QPSK signal passes through zero amplitude (the center of the constellation diagram). As for BPSK, this is undesirable. One solution is *offset QPSK* or OQPSK. In OQPSK we simply offset one of the bit streams $m_I(t)$ or $m_Q(t)$ by ½ a symbol period. With this offset they transition at different times, so we can never have both pass through zero at the same time. With this offset, the signal of Fig. 23.4 becomes that of Fig. 23.5. There are still some amplitude fluctuations, but they are much smaller than for filtered QPSK, and most importantly the signal never approaches zero.

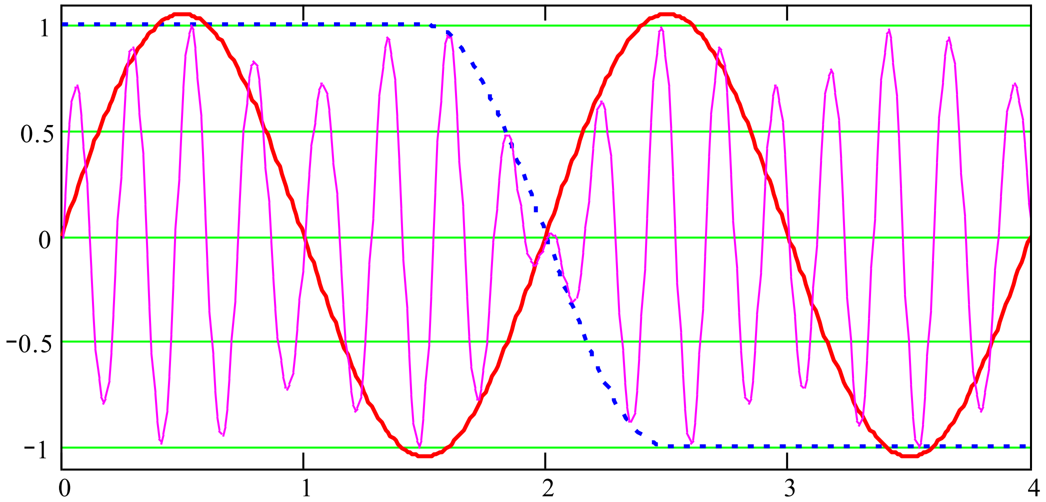

Figure 23.4: Filtered QPSK. The phase transitions are smoothed but amplitude fluctuations are introduced.

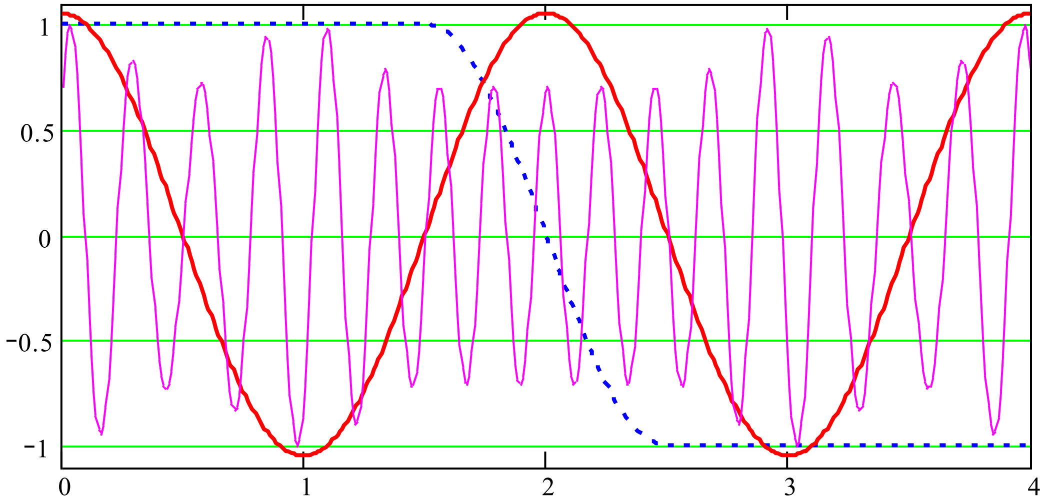

Figure 23.5: Filtered OQPSK. The phase transitions are still smooth, but because $m_I(t), m_Q(t)$ *do not transition at the same time, the signal never passes through zero amplitude (the center of the constellation diagram).*

OQPSK is used on the uplink (reverse channel) of the IS-95 CDMA system in the US. Why is OQPSK used on the uplink while straight QPSK is used on the downlink? On the downlink, the base station sums together several QPSK signals, one for each mobile plus control channels and so on. It is this sum of signals that gets amplified. On the other hand, on the uplink, only a single signal is being sent. Therefore, the amplitude variation problem illustrated in Fig. 23.4 comes into play.

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