BPSK

Introduction

Recall that the most general modulation has the form $s(t) = a(t)\cos[\omega_c t + \phi(t)]$. We remarked earlier that phase modulation was not an effective way to implement analog communication, one reason being because it requires a phase reference. For digital systems this problem can be overcome. Phase modulation is an effective digital communication strategy, and we refer to it as *phase shift* keying or PSK. In PSK we limit ourselves to discrete values of the phase, $\phi = \phi_k$ so that we have a discrete set of RF signals

$$s_k(t) = A_c \cos(\omega_c t + \phi_k) \tag{22.1}$$

We use the different phases to represent different logic states. We can rewrite (22.1) in the form

$$\begin{aligned} s_k(t) &= A_c\left[\cos\phi_k \cos\omega_c t - \sin\phi_k \sin\omega_c t\right] \\ &= A_c\left[a_k \cos\omega_c t - b_k \sin\omega_c t\right] \end{aligned} \tag{22.2}$$

which shows that phase shift keying signals are linear combinations of $\cos\omega_c t$ and $\sin\omega_c t$.

For *binary phase shift keying*, or BPSK, we use $\phi = 0$ to represent a logic "1" and $\phi = \pi$ to represent logic "0". In this case we have simply

$$\begin{aligned} s_1(t) &= A_c \cos(\omega_c t) \\ s_0(t) &= A_c \cos(\omega_c t + \pi) \\ &= -A_c \cos(\omega_c t) \end{aligned} \tag{22.3}$$

Since a phase shift of π just changes the sign of the carrier, we can write $s(t) = A_c m(t)\cos(\omega_c t)$ where $m(t) = \pm 1$, and $+1$ represents logic "1", and -1 represents logic "0". Thus a BPSK signal has the form of an AM signal. This is the most useful point of view, particularly when filtering is applied to reduce the bandwidth. An example BPSK signal is shown in Fig. 22.1.

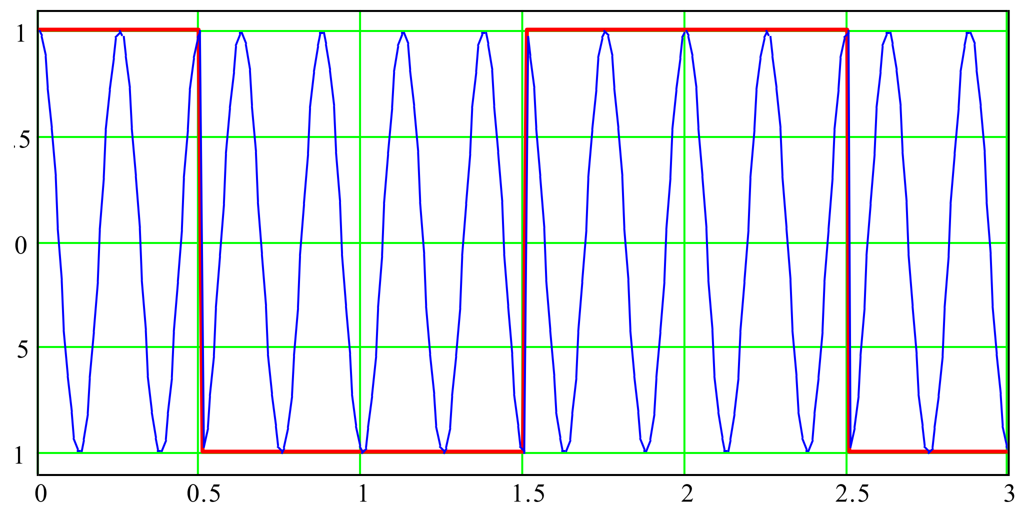

Figure 22.1: BPSK signals. The modulation is $m(t) = \pm 1$ and the phase transitions are sharp.

Constellation Diagram

As shown above, phase shift keying signals are linear combinations of $\cos\omega_c t$ and $-\sin\omega_c t$. We can represent one of these signals as a point in Cartesian coordinates where the horizontal coordinate gives the coefficient a_k and the vertical coordinate b_k. The result is called a *constellation diagram.* It is simply a phase diagram where amplitude and phase are given by A_c and ϕ. The constellation diagram for BPSK is illustrated in Fig. 22.1.

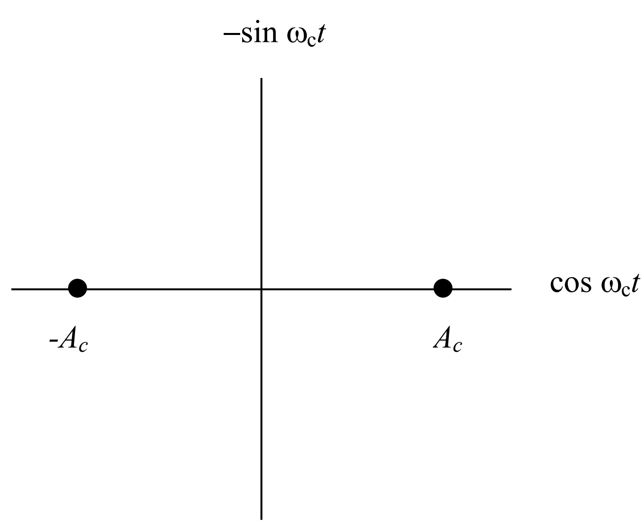

Figure 22.2: Constellation diagram for BPSK. The black dots denote the phasors of the ideal signals that represent logic states "0" and "1".

BPSK Demodulation

Previously we noted that phase modulation suffered from the fact that phase is a relative concept, so to perform demodulation a receiver would need to know the phase of the carrier used in the transmitter. With BPSK the carrier can be obtained by a process called *carrier recovery.* A BPSK signal is proportional to ± 1 times $\cos \omega_c t$. Since $(\pm 1)^2 = 1$, if we square a BPSK signal the modulation disappears and we get $\cos^2 \omega_c t = [1 + \cos 2\omega_c t]/2$. This is a pure sinusoid at twice the carrier frequency (plus some DC). Using "phase lock loop" circuits and a frequency divider, the carrier $\cos \omega_c t$ can be "recovered" from this.

Assuming we have recovered the carrier, we can demodulate a BPSK signal by calculating

$$\begin{aligned} r &= \frac{2}{T_b}\int_0^{T_b}\left[A_c m \cos \omega_c t + n(t)\right]\cos \omega_c t \, dt \\ &= A_c m + \frac{2}{T_b}\int_0^{T_b} n(t)\cos \omega_c t \, dt \\ &= A_c m + X \end{aligned} \tag{22.4}$$

where m is either ± 1 and X is a Gaussian RV with zero mean and variance $\sigma_n{}^2 = N_0 / T_b$. If $r > 0$ we assume $m = 1$, i.e., that a logic "1" was sent. Otherwise we take $m = -1$, i.e., logic "0". Say $m = -1$. Then we make an error if $X > A_c$ because that will cause r to be greater than zero. The probability of this is

$$\begin{aligned} P_e &= \frac{1}{\sqrt{2\pi}\sigma_n}\int_{A_c}^{\infty} e^{-\frac{1}{2}\left(\frac{x}{\sigma_n}\right)^2} dx \\ &= Q\left(\frac{A_c}{\sigma_n}\right) \\ &= Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \end{aligned} \tag{22.5}$$

Compare this to (21.11) for FSK. Here we get an additional factor of 2. This means that with half the power BPSK can give the same BER as FSK. Therefore, BPSK is more power efficient than FSK. In fact we can say it is a factor of 2, or 3 dB, more efficient.

BPSK Spectrum

Recalling that our BPSK signal is $s(t) = A_c m(t)\cos(\omega_c t)$, we can use the convolution theorem to calculate its spectrum. The spectrum of the product of two signals is the convolution of their individual spectra. The spectrum of $\cos(\omega_c t)$ is $[\delta(f - f_c) + \delta(f + f_c)]/2$, so

$$S(f) = \frac{A_c}{2} M(f - f_c) + \frac{A_c}{2} M(f + f_c) \tag{22.6}$$

where $M(f)$ is the Fourier transform of $m(t)$. To calculate this you'd need to know the specific sequence of logic states. If you assume the bits are random, then you can show that the power spectrum of $m(t)$ is identical to the power spectrum of a single bit. Hence we have

$$|S(f)|^2 = T_b \operatorname{sinc}^2[T_b(f - f_c)] \tag{22.7}$$

where we've limited ourselves to positive frequencies and normalized the power spectrum to have unit energy. This is illustrated in 22.3 and compared to the MSK spectrum.

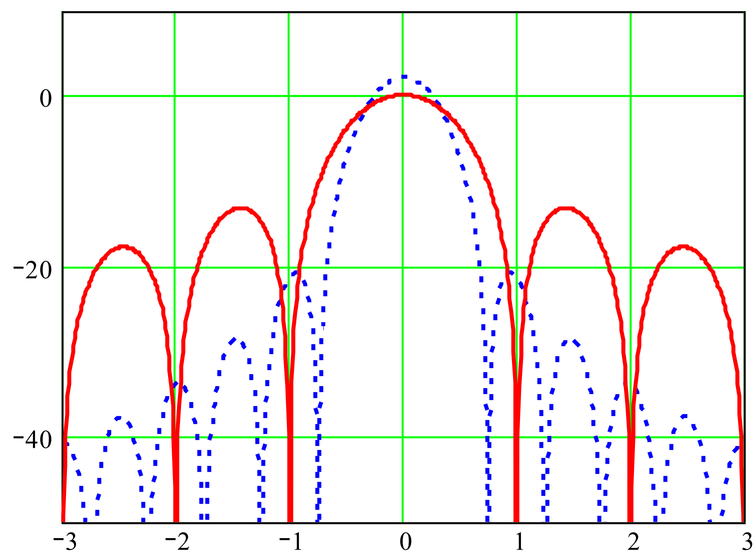

Figure 22.3: BPSK spectrum (solid red curve) compared to MSK spectrum (dashed blue curve). Both curves have unit energy (on linear scale). Horizontal axis is normalized frequency fT_b relative to the carrier, vertical axis is amplitude in dB.

The wider main lobe of BPSK as compared to MSK, and the much larger side lobes show that BPSK is not as bandwidth efficient as MSK. We can quantify bandwidth efficiency as follows

$$\eta = \frac{R_b}{B_{RF}} \left(\frac{bits/s}{Hz} \right) \quad (22.8)$$

where R_b is the bit rate and B_{RF} is the RF bandwidth. This, of course, depends on how we define the bandwidth. And, in practice, these modulation techniques are usually accompanied by filtering to reduce the side lobes. If we take the frequency spread between the nulls of the main lobe as an effective bandwidth, then we find that the spectral efficiency of BPSK is $\eta = 0.50$ while that of MSK is $\eta = 0.67$. So, we get 34% more bits per second through an MSK channel than through a BPSK channel with the same bandwidth.

The side lobes of the sinc function (Fig. 22.3) are very large and is left unfiltered they could cause interference with adjacent RF channels. Therefore filtering is typically applied. This smoothes the rectangular pulses that make up $m(t)$. The result for a single pulse is shown in Fig. 22.4.

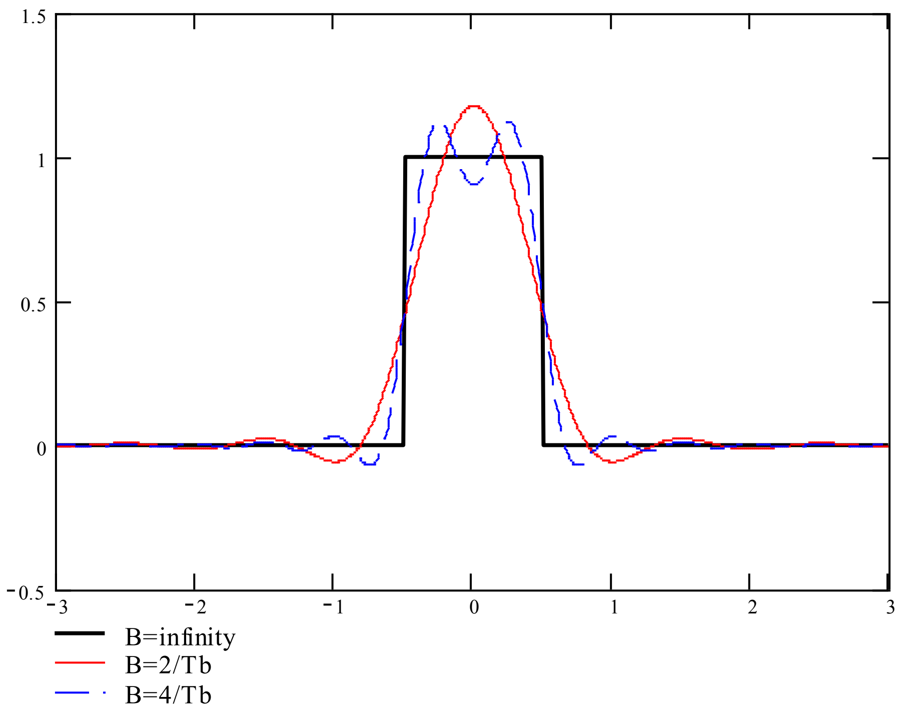

Figure 22.4: Rectangular pulse and filtered versions. Horizontal axis is time in units of T_b. Solid (red) curve corresponds to elimination of all sidelobes of the BPSK spectrum. Dashed (blue) curve corresponds to keeping only the first sidelobes.

This has the effect of smoothing the bit transitions as shown in Fig. 22.5.

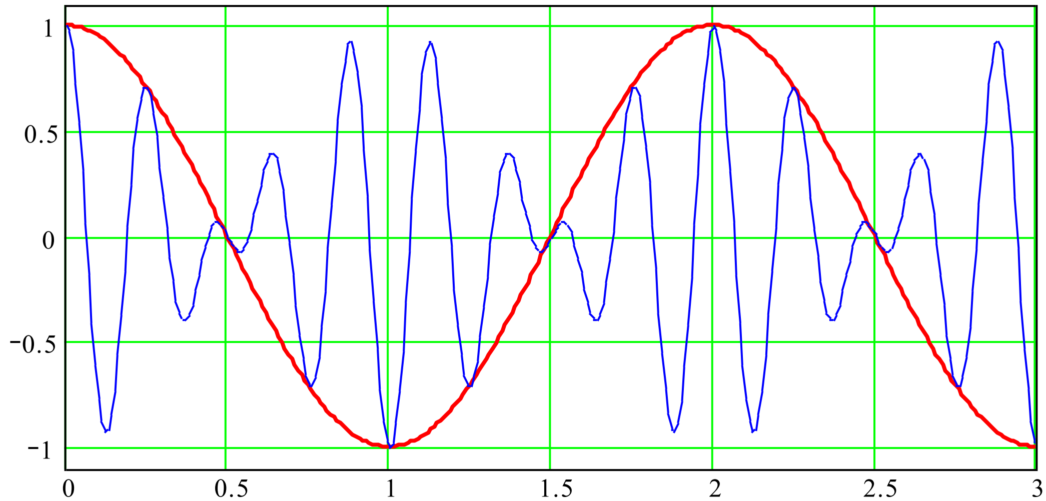

Figure 22.5: Filtered BPSK. The bit transitions are smoothed, but this introduces amplitude fluctuations.

The filtering, while reducing side lobes and hence total bandwidth, introduces amplitude fluctuations. This can cause a problem if the signal passes through a non-linear RF power amplifier, and all real amplifiers are to some extent non-linear. The resulting distortion can lead to *spectral regrowth* that creates unwanted signals outside the desired bandwidth. This is a relative disadvantage of BPSK as opposed to FSK. With FSK the signal always has constant amplitude, even when the filtering is modulated.

References

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2. Proakis, J. G. and M. Salehi, *Communication Systems Engineering, 2nd Ed.,* Prentice Hall, 2002, ISBN 0-13-061793-8.