Frequency Reuse and Cellular Radio

Introduction

Let's begin by considering some different strategies for wireless communication.

"Pre-Cellular" Radio Communication

Let's start with the "CB" (Citizen's Band) model of wireless communication. Prior to the days of cell phones (1983 in the US), having a CB or amateur (HAM) radio in your car was about the only option for mobile communication. CB radios, which use AM at about 27MHz and are limited to 4 watts of transmitter power, are still very popular with truckers, among others. More recently the FCC opened a new public band, the FRS (Family Radio Service) at around 460Mhz. FRS radios use FM and are limited to ½ watt transmitter power.

Consider the situation shown in Figure 13.1. Here users A, B, and C want to be able to communicate with each other at will. Every radio has access to some number of channels (40 for CB radios). All users monitor the same "contact" channel (typically channel 19 for CB). If A wants to talk to B, she transmits something like "This is A calling B." If B receives this he comes back with something like "This is B, go ahead A." They can then converse. They might agree to switch to another channel so as to free up the contact channel for others. When they're done they switch back to the contact channel and continue monitoring.

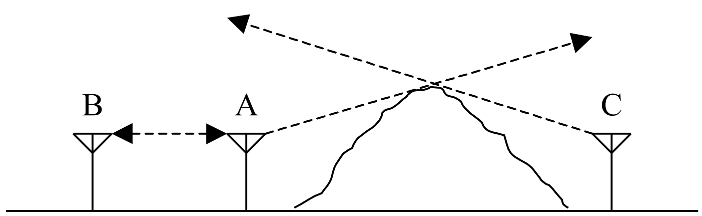

Figure 13.1: The CB model of wireless communication.

A big advantage of this approach is the low cost and simplicity. The "CB network" is unsupervised and no central infrastructure is required; anyone can join the network by purchasing a radio (which you can get for around $30) and you don't have to pay for the service or airtime.

There are some disadvantages, however. The biggest is that users have to have a good radio path between them to communicate. For example, if A wants to talk to C and there is a mountain between them, it probably isn't going to happen. Or if A and B are too far apart, they won't be able to communicate. If they are moving, they may make contact only to eventually loose it when they get too far apart.

Another major disadvantage is that you have to constantly monitor lots of other people's conversations while waiting for someone to contact you, i.e., the network isn't able to "ring" your radio when someone wants to talk to you specifically. A related disadvantage is that there is no privacy. Anyone with a radio nearby can hear your conversation. Also, there is nothing to keep another user from trying to transmit on the same channel at the same time and "stepping on" your conversation.

The "repeater model" solves the radio-path problem by putting a "repeater" radio (R) at a high point (Figure 13.2). The "mobiles" (A, B, and C) transmit to the repeater and the repeater "repeats" that transmission to all mobiles. A repeater is essentially two radios with one in receive mode at frequency f_1 and the other in transmit mode at frequency f_2. The audio output of the receiver becomes the audio input of the transmitter. (Two frequencies are needed, otherwise the repeater would repeat its own transmission.) In this scenario mobiles need only have a good radio path to the repeater in order to communicate with one another, e.g., A can talk to C just as easily as to B. Most police and fire department radio systems use repeaters of this sort.

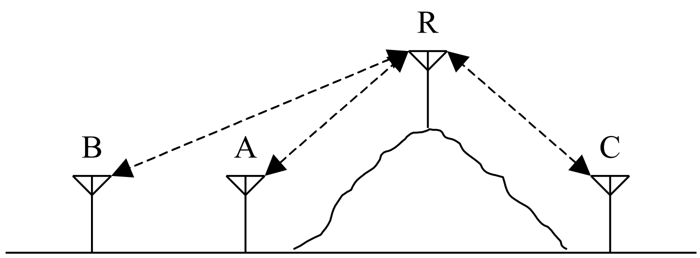

Figure 13.2: The repeater model of wireless communication. Mobiles A, B, and C communicate through the repeater R.

Although the radio path problem is largely solved, the other problems mentioned above remain. And, an obvious new disadvantage is that infrastructure is required, namely the repeater, and someone has to pay for this. Radio clubs often pay for amateur radio repeaters. For example, in the Pullman area KBARA (Kamiak Butte Amateur Repeater Association) maintains a repeater on top of Kamiak Butte.

Imagine now that there is a person at the repeater with a phone. He's listening to all the conversations. "A" gets on her radio and says, "I'd like to make a phone call to 555-1212, please." "R" then dials 555-1212 on his phone. He then plugs the audio out of the repeater into the audio in of the phone and the audio out of the phone into the audio in of the repeater. Now, anything said by A on her radio (or by B or C, for that matter) will go over the phone line, and anything said by the person at 555-1212 will be transmitted by the repeater and heard by A (and B and C). Conversely, suppose someone calls the person at the repeater on the phone and says, "I'd like to speak to A, please." The repeater guy could then get on his radio and call for A. If he gets hold of her he could then "patch" in the phone line like before. In this way phone calls can be made to and from mobile radios, and such services were available long before there were cell phones available (e.g., commercial fishing boats could call a "Marine Operator" to patch in a phone call). If everyone's radio has a phone-like, touch-tone keypad, the whole process can be automated at the repeater. The result is an "autopatch." Many amateur radio repeaters have autopatch capability.

A repeater system still requires every user to be close enough to communicate with the repeater, so it forms local network. An extension of this is the linked-repeater concept (Figure 13.3). Here two or more repeaters are placed at various locations. There are radio links (or possibly fiber optic or copper wire links) between repeaters so that every repeater retransmits any transmission from a radio anywhere in the system. With a system like this you can have radio communication over an arbitrarily large area. For example, the KBARA repeater on Kamiak Butte is occasionally linked in to the "Evergreen Intertie" network resulting in radio coverage over most of the state of Washington.

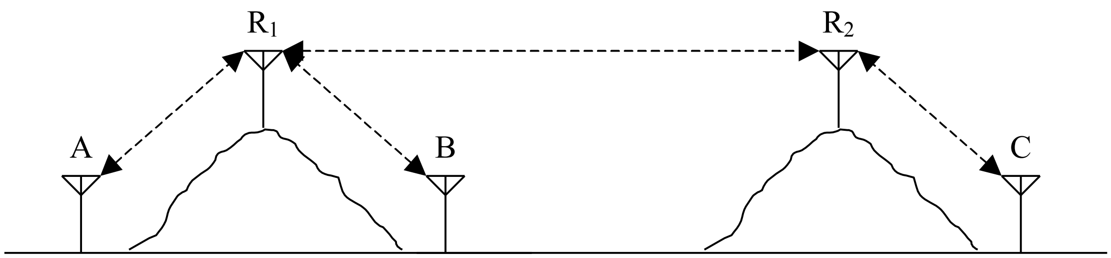

Figure 13.3: The linked-repeater approach to wireless communication.

A system of this sort is great for the amateur radio hobby, but it still has some serious drawbacks if you wanted to use it as a cellular phone system. The biggest is that the system can only carry one conversation at a time! The cellular phone systems that we will study are an extension of the linked-repeater concept. Those extensions are what make the difference between a network appropriate for causual/hobby use and a commercial network serving millions of people.

Frequency Reuse

Suppose that a wireless phone service uses a certain bandwidth B_T of the radio spectrum. If each phone channel requires a bandwidth B_C then there are $C = B_T / B_C$ total channels available for this service. This is a fixed resource and we want to make the most effective use of it. This leads to the idea of *frequency planning* to achieve maximum possible *frequency reuse*. At a given base station only a single customer can use a given channel. However at another base station sufficiently far away, propagation losses might be large enough to effectively isolate the stations thereby allowing this same channel to be *reused* at the other station. The more a system can reuse channels, the more customers can be served, and the more revenue can be generated.

Hexagonal Geometry

If the earth were a uniform flat surface, contours of constant received power would be circles centered at the transmitter. When more than one transmitter is operating we are often interested in determining which transmitter will provide the strongest signal at a given point on the earth. Assuming identical transmitters, this will simply be the closest transmitter to the receiver. Since the locus of points that are equidistant from two given points is a line, boundaries where the received power is the same from two transmitters will be linear. As shown in Fig. 13.1 this leads polygonal "cells" around each transmitter.

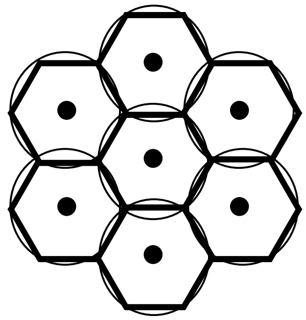

Figure 13.1: Dots represent transmitters. Constant power contours are overlapping circles. Hexagonal "cells" represent regions that are closer to "their" transmitter than to any others.

The hexagon is the standard cell shape assumed for cellular telephone systems. As shown in Fig. 13.2, if the maximum radius of the hexagon is R, then the minimum radius is $\sqrt{3}R/2$. The area of the cell can be calculated as the sum of the areas of six equilateral triangles to get $A = 3\sqrt{3}R/2 \approx 2.6R^2$.

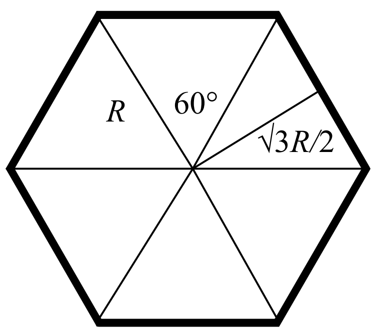

Figure 13.2: Hexagonal geometry of a radio cell. Cell area is $A = 3\sqrt{3}R/2 \approx 2.6R^2$.

Reuse Patterns

Basically, frequency planning involves covering a region of the map with a tiling of hexagonal cells and then assigning groups of channels to each cell. We will describe our frequency reuse pattern as follows. Choose any integers j and k. Starting at the center of a particular cell, move perpendicular to one of the six sides a distance of j cells. Now turn left 60° and move a distance of k cells. The new cell you end up in is to use the same set of channels as the original cell. Because there are six sides to the hexagon, there are six different *first-tier co-channel* cells defined by this process. See Fig. 13.3

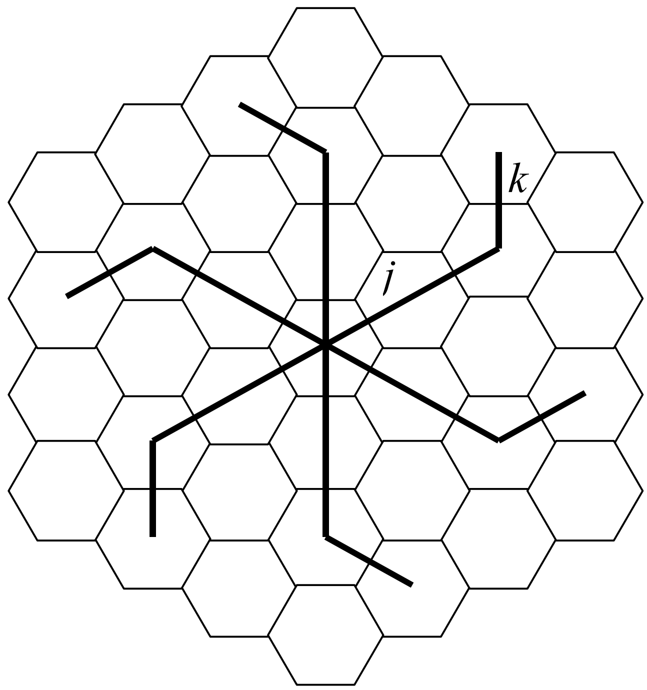

Figure 13.3: A frequency reuse pattern defined by two integers j and k. In this case j is 2 and k is 1. This locates 6 co-channel cells.

Let's consider how we could create a reuse pattern with this technique. To begin, assume we assign some set of channels called "A" to the center cell in Figure 13.3. Let's take $j = k = 1$. Move in the manner described above in the six possible directions to locate the six nearest co-channel cells. Label them as "A." Then find the six nearest co-channel cells to those and label them as "A," and so on. We arrive at the pattern shown in the left of Fig. 13.4.

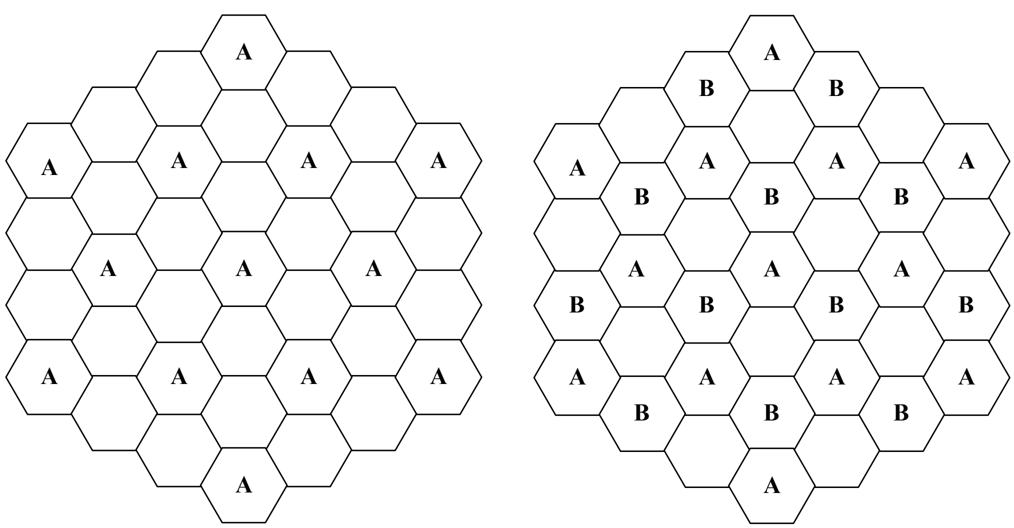

Figure 13.4: Locating co-channel cells.

The blank cells cannot be reached from an "A" cell by a $j = k = 1$ move. We will have to assign new frequencies to these blank cells. Choose one of them. Let's say we choose the one above the central "A" cell. Label this "B." This corresponds to a new group of frequencies/channels. Now repeatedly perform $j = k = 1$ moves until you've filled all the blank cells you can. We arrive at the situation shown in the right of Fig. 13.4. There are still some blank cells and we will need a new set of channels for them. We continue in the manner until all cells are filled.

N=3 Reuse Pattern: j=1, k=1

If we take one more step in the process shown in Fig. 13.4 using a channel set called "C" we fill all cells with either "A", "B", or "C". This is shown in Fig. 13.5. Notice how the pattern can be thought of as consisting of repeated *clusters* of three cells: an "A", a "B", and a "C" cell. We call this an $N = 3$ *reuse pattern* because it requires three different sets of channels. If we assign the same number of channels to each set, then we will have $1/3$ of the total available channels operating in each cell. We say we have a *reuse factor of* $1/3$.

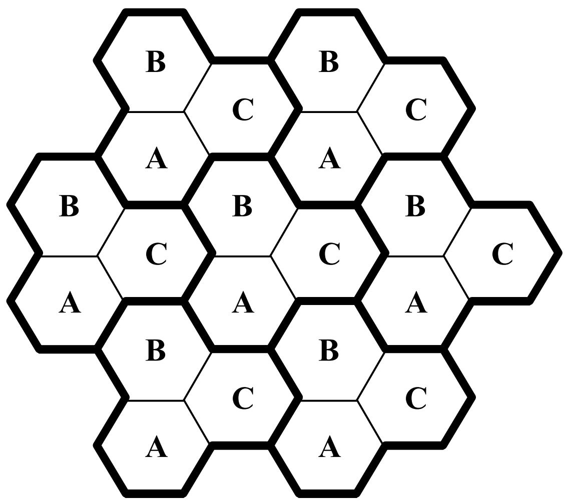

Figure 13.5: $N = 3$ *reuse pattern.*

The geometrical relationship of co-channel cells is important because these are the cells that cause interference with one another. Extracting an "A" cell and its six nearest co-channel cells from the $N = 3$ reuse pattern, we get the situation diagrammed in Fig. 13.6

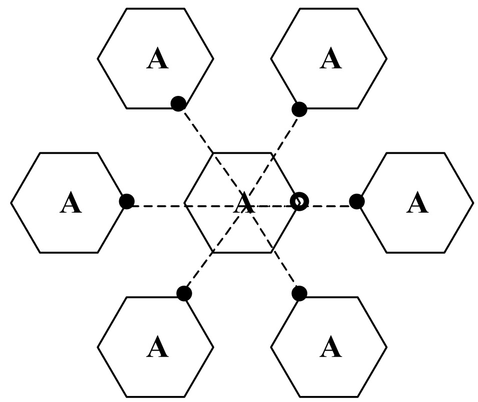

Figure 13.6: Worst-case first-tier uplink interferers for N=3 pattern.

Here we've also indicated mobile transmitters located so as to produce the worst-case S/I. The open circle represents the mobile that produces the signal that the center "A" base station is trying to receive. When it is as far from the base station as possible it will produce the weakest signal. The other six mobiles produce the interference. When they are as close as possible to center base station they produce the largest interference. Hence the case shown is the worst case for S/I. If the propagation exponent is *n*, then on a linear scale power is proportional to $1/r^n$. If the signal mobile is r_S from the base station and the interfering mobiles are r_I, then

$$S/I = \frac{\frac{1}{r_S^n}}{6\frac{1}{r_I^n}} = \frac{1}{6}\left(\frac{r_I}{r_S}\right)^n \tag{13.1}$$

The factor of 6 in the denominator accounts for the 6 interferers. Since this expression depends on the ratio of distances, it is independent of the absolute size of the cells, e.g., if we double the size of the cells then both r_S and r_I will double and their ratio will stay the same. So S/I depends only on the reuse pattern and the propagation exponent *n*. Since $r_I/r_S > 1$, we see that S/I increases with increasing *n*. This makes sense. The larger *n* is, the faster received power falls off with distance. Since the interference mobiles are farther away than the signal mobile, the interference power will decrease relatively more than the signal power as *n* increases.

With a little trigonometry you can find that $r_I = 2r_S$, so

$$S/I = \frac{2^n}{6} \tag{13.2}$$

is the worst case S/I. For $n = 4$ ("typical" of large cities), $S/I = 2.7 = 4.3dB$, and the signal is not even 3 times as strong as the interference. For the free-space value $n = 2$, $S/I = 0.67 = -1.8dB$, and the signal is actually weaker than the interference!

Hopefully the worst-case will rarely occur. It is not likely that all the interferers will be as close as possible to the base station at the same time. A more "average" case is illustrated in Fig. 13.7. Here we assume the interferers are at the center of their cells.

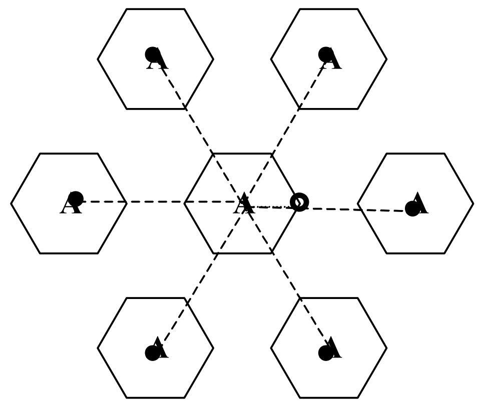

Figure 13.7: "Average-case" first-tier uplink interferers for N=3 pattern.

In this case $r_I = 3r_S$ and

$$S/I = \frac{3^n}{6} \tag{13.3}$$

For $n = 4$, $S/I = 13.5 = 11dB$. For $n = 2$, $S/I = 1.5 = 1.8dB$.

N=4 Reuse Pattern: j=2, k=0

If we define a reuse pattern by $j = 2$, $k = 0$ we get the situation shown in Fig. 13.8 in which we need four sets of channels. Hence this is an $N = 4$ reuse pattern with a reuse factor of $1/4$.

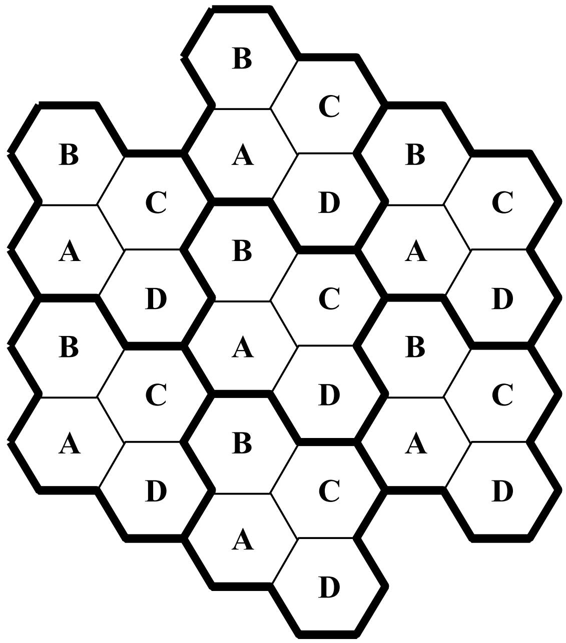

Figure 13.8: N=4 reuse pattern.

The worst-case first-tier interferers is shown in Fig. 13.9. With some trigonometry we find $\frac{r_I}{r_S} = \frac{3\sqrt{3}}{2}$, so

$$S/I = \frac{1}{6}\left(\frac{3\sqrt{3}}{2}\right)^n \quad (13.4)$$

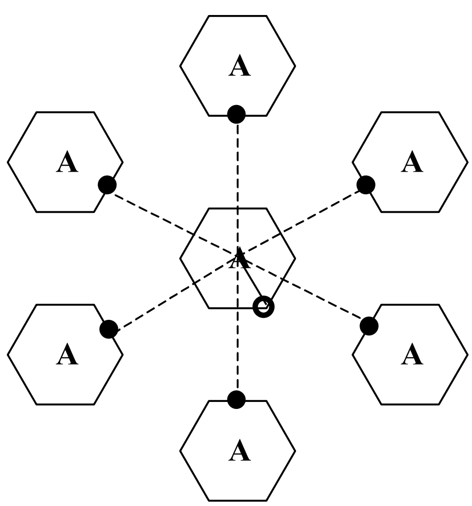

Figure 13.9: First-tier interferers for N=4 pattern.

Worst case for $n = 4$ is $S/I = 7.6 = 8.8dB$. For $n = 2$, $S/I = 1.1 = 0.5dB$. For the "average" case with the mobiles at the center of their cells $r_I / r_S = \sqrt{12}$ and

$$S/I = \frac{\left(\sqrt{12}\right)^n}{6} \tag{13.5}$$

With $n = 4$ we have $S/I = 24 = 14dB$. With $n = 2$ we have $S/I = 2 = 3dB$. These examples show how strongly S/I depends on the propagation exponent. We see that rapidly decaying fields are a *good* thing for S/I.

N=7 Reuse Pattern: j=2, k=1

A reuse pattern with $j = 2, k = 1$ results in the situation shown in Fig. 13.10. We need 7 channel sets in this case so $N = 7$ and the reuse factor is $1/7$.

There are no values of *j* and *k* that give $N = 5$ or $N = 6$. In fact it can be shown that

$$N = j^2 + k^2 + jk \tag{13.6}$$

so the only possible values of *N* are 3, 4, 7, 9, 12, and so on.

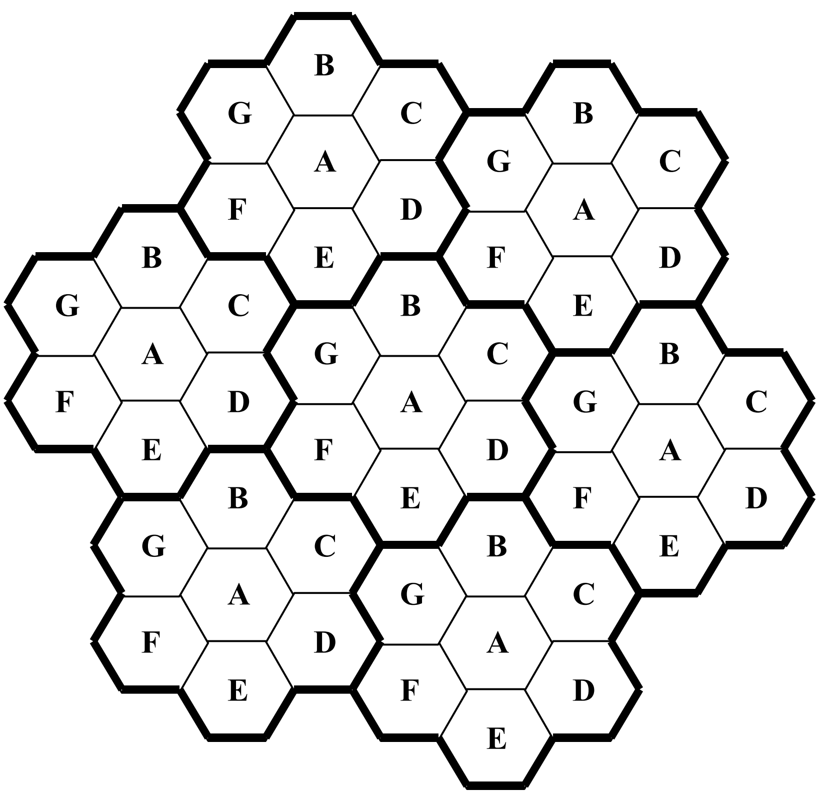

Figure 13.10: N=7 reuse pattern.

The worst case first-tier interferers is shown in Fig. 13.11.

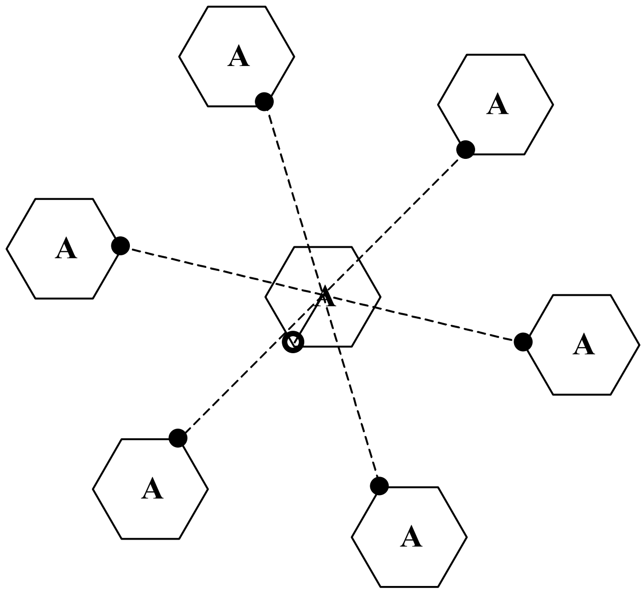

Figure 13.11: Worst-case first-tier interferers for N=7 pattern.

Trigonometry gives $r_I / r_S = \sqrt{13}$ so

$$S/I = \frac{\left(\sqrt{13}\right)^n}{6} \tag{13.7}$$

With $n = 4$ we have $S/I = 28 = 14.5dB$. With $n = 2$ we have $S/I = 2.2 = 3.4dB$. The average case has $r_I / r_S = \sqrt{21}$ so

$$S/I = \frac{\left(\sqrt{21}\right)^n}{6} \tag{13.7}$$

With $n = 4$ we have $S/I = 73.5 = 18.7dB$. With $n = 2$ we have $S/I = 3.5 = 5.4dB$.

You can show that for any reuse factor $1/N$ the "average" S/I case has

$$\frac{r_I}{r_s} = \sqrt{3}\sqrt{j^2 + k^2 + jk} = \sqrt{3N} \tag{13.8}$$

from which

$$S/I = \frac{\left(\sqrt{3N}\right)^n}{6} \tag{13.9}$$

We see that *S/I* increases with *N*. This is because the larger *N* is the larger the separation of the co-channel cells. *S/I* also increases with *n*. This is because larger propagation exponents lead to a more rapid decay of fields. Since the interferers are always farther from the basestation than the signal source, the interference decreases more rapidly with increasing *n* than the signal.

	S/I (dB)	
N	$n=2$	$n=4$
3	2	11
4	3	14
7	5	19

Figure 13.12: "average" S/I for different reuse patterns and propagation exponents.

Figure 13.12 gives *S/I* for various values of *N* and *n*.

References

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2. Rappaport, T. S., *Wireless Communications: Principles and Practice*, Prentice Hall, 1996, ISBN 0-13-375536-3.