S/N and S/I

Introduction

Up to now we have spent quite a bit of effort learning how to predict received signal strength. There are two other sources of power in a receiver: noise and interference. Noise power arises within the receiver itself and is the result of random thermal motion of charge carriers. Interference arises when other radios are simultaneously using the same channel. Since the number of radio channels is a limited resource, to allow a large number of people to use a cellular system we have to *reuse* channels (referred to as *frequency reuse* – we'll consider this topic later) so we'll almost always have interference present.

Signal-to-Noise Ratio (SNR or S/N)

Imagine connecting an ideal oscilloscope to a resistor. You might expect the scope to show a flat line at 0 volts since there is no external source driving current through the resistor. However, the resistor has some temperature T which means that the electrons in it are randomly bouncing around with a kinetic energy proportion to T. Since moving electrons create a current, this means that there is a random current, and hence voltage, in the resistor. So in fact your scope will show a random waveform, i.e., noise, which will increase in amplitude with an increase in temperature. This thermal noise will act as a random power source in a radio receiver, generating power at all frequencies.

An analysis of the thermal physics shows that the total noise power present in a receiver is

$$P_{N,W} = kT_N B \tag{12.1}$$

where $k = 1.38 \cdot 10^{-23} \text{JK}^{-1}$ (joules per degree kelvin) is *Boltzmann's constant*, T_N is the *noise temperature* of the receiver (which is not necessarily the same thing as the physical temperature), and B is the bandwidth of the receiver (usually equal to the bandwidth of the received channel). We see that noise power goes up as the bandwidth does. This makes sense because increasing the bandwidth means that we'll be "tuned" into more thermal noise sources. Noise power also goes up as the noise temperature does. Again this makes sense because higher temperature means more vigorous random motion of the charges.

Some quantities related to noise temperature are the *noise factor* (F), defined as $F = 1 + T_N / 290K$, and *noise figure* (NF) defined by $NF = 10\log F$. T_N, F, and NF are just different ways to represent the same thing. Figure 12.1 shows F and NF for various noise temperatures and the corresponding noise power for 1kHz bandwidth.

			B=1kHz
Tn(K)	**F**	**NF (dB)**	**Pn(dBm)**
35	1.12	0.5	-153
75	1.26	1.0	-150
290	2.00	3.0	-144
2610	10.00	10.0	-134
8881	31.62	15.0	-129

Figure 12.1: Noise figure, noise factor, and noise power into 1kHz bandwidth for various noise temperatures.

NF is usually used to characterize receivers. Lowering *NF* lowers receiver noise but also costs money. An inexpensive receiver might have a *NF* of 10dB. A “pretty good” receiver might have a *NF* of 3dB giving 10dB less noise power. A “really good” (“low noise”) receiver might have a *NF* of 0.5dB giving another 10dB less noise.

Signal-to-noise ratio (SNR or *S/N*) is simply the ratio of signal power to noise power:

$$S/N = \frac{P_{R,W}}{P_{N,W}} \qquad S/N_{dB} = P_{R,dBm} - P_{N,dBm} \tag{12.2}$$

It measures how strong the received signal is relative to the thermal noise in the receiver. If *S/N* is large it means that the noise power is relatively insignificant. On the other hand, if *S/N* is small then the noise power begins to “compete” with the signal power. In an analog system this will mean that the audio will be noisy. In a digital system it means that bit errors will be generated.

Analog and digital demodulation schemes typically have a minimum *S/N* required for proper operation. For example, FM radio starts to sound pretty poor if *S/N* drops below 10dB, and a *S/N* of 20dB or more is desirable. A cellular system designer is usually stuck with given noise powers in the receivers. Therefore her task is to make sure that the received signal power is large enough to generate an acceptable *S/N*.

Example 12.1

Assume received signal strength is given by $P_{R,dBm} = -40 - 35\log(r/100\text{m})$. Your receiver has $NF = 10\text{dB}$ and a channel bandwidth $B = 30\text{kHz}$. At what distance does the *S/N* equal 10dB?

From Figure 12.1, a 10dB *NF* gives –134dBm noise power into 1kHz. Since $10\log 30/1 = 14.8\text{dB}$, a 30kHz bandwidth will have a noise power 14.8dB greater than this, or $-134\text{dBm} + 14.8\text{dB} = -119\text{dBm}$. If the *S/N* is 10dB, then the signal power must be $-119\text{dBm} + 10\text{dB} = -109\text{dBm}$. Solving $-40 - 35\log(r/100\text{m}) = -109$ gives $r = 9.4\text{km}$.

Consider the downlink/uplink situation in Figure 4.3 and Equation (4.6). Assume the noise powers in the mobile and basestation are P_{NM} and P_{NB}. Then the *S/N* values are (all values are on a dB scale)

$$\begin{aligned} S/N_M &= P_{RM} - P_{NM} = P_{TB} - P_{NM} - L \\ S/N_B &= P_{RB} - P_{NB} = P_{TM} - P_{NB} - L \end{aligned} \tag{12.3}$$

If we want these to be the same, so that both links have the same quality, we require $P_{TB} - P_{NM} = P_{TM} - P_{NB}$. Also, we want this quantity to be large enough to provide good *S/I* even in presence of significant system loss L. Consider the mobiles. These are consumer products, so price is an issue. Also, they're battery powered, so transmitter power is necessarily limited. This means that it is difficult to push P_{NM} down much or push P_{TM} up much. Base stations, on the other hand, typically cost tens or hundreds of thousands of dollars, so the price of a good low-noise receiver is relatively insignificant. Also, they have access to wired power so transmitter power is not hard to come by. Therefore, typically we would push $P_{TB} - P_{NM}$ up by increasing basestation transmitter power and push $P_{TM} - P_{NB}$ up by reducing basestation noise figure. In addition, as we discussed previously, we would probably also use diversity reception at the base station to improve things even further.

In principle, we can achieve arbitrarily high *S/N* by increasing transmitter power and/or reducing receiver noise – at a cost, of course. However, the effect of interference from other mobiles operating on our channel is more fundamental. We now consider this.

Signal-to-Interference Ratio (S/I)

There is a finite amount of radio spectrum available for telecommunications and we need to use that limited resource efficiently. If, for example, at any given time on any given radio frequency we only allowed a single cell phone call in the entire United States, cellular phone systems could not support enough users to be economically viable or of much use to society. Instead we employ *frequency reuse* whereby we allow several communication links to operate at the same frequency, provided that they are geographically "sufficiently far apart." Figuring out what sufficiently far apart is requires us to develop the idea of the signal-to-interference radio (S/I). We'll consider frequency reuse in a future lecture.

Consider the situation shown in Fig. 12.1. (For this discussion we will measure all power on a linear scale.) Here basestation B_1 is transmitting to mobile phone M_1 and basestation B_2 is transmitting to mobile phone M_2. Both links use the same radio frequency. M_1 will receive two signals. The desired signal from B_1 will have some power level P_{11}. But there will also be an undesired signal received from B_2 with power level P_{12}. We refer to this undesired signal as *interference* and call the ratio P_{11}/P_{12} the *signal-to-interference ratio*, or *S/I*. M_1 doesn't know which of these transmissions is signal and which is interference, and in fact as far as the radio receiver is concerned there is only one signal – the sum of the desired and undesired transmissions. However, if *S/I* is large enough the desired signal will dominate the receiver and M_1 will only hear the transmission from B_1.

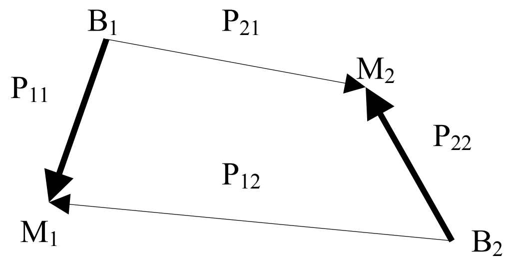


Figure 12.1: Geometry of co-channel interference.

However, if *S/I* is small, then the interference signal becomes significant relative to the desired signal and this will degrade our reception.

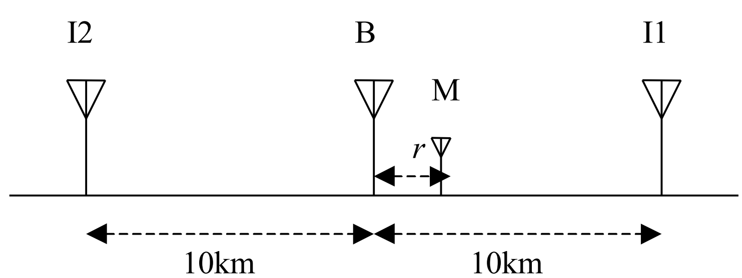
If there are *N* phones using the same channel, then the *S/I* will be

$$S/I = \frac{P_{11}}{P_{12} + P_{13} + \cdots P_{1N}} \tag{12.4}$$

Remember that this formula is only valid if the powers are on a linear scale – you can't add powers on a logarithmic scale.

Example 12.2

Consider the situation illustrated below. A mobile (M) is trying to receive a signal from base station B. Two other base stations, I1 and I2, are 10km away and using the same frequency. Assuming received signal strength from any base station is given by $P_{R,dBm} = -40 - 35\log(r/100\text{m})$, how far from B can M be and still have *S/I* of at least 12dB?



Call $r_1 = 10\text{km} - r$ and $r_2 = 10\text{km} + r$ the distances from I1 and I2 to M. On a linear scale $P_R \propto r^{-3.5}$ $(10n = 35)$. So

$$S/I = \frac{r^{-3.5}}{(10-r)^{-3.5} + (10+r)^{-3.5}}$$

with r in km. On a linear scale 12dB is $10^{12/10} = 15.8$. Solving for the r that gives $S/I = 15.8$ (e.g., using a programmable calculator, Matlab, or Mathcad) we find $r = 3.06\text{km}$.

S/I is a more fundamental limitation in a cellular network because you cannot improve it with improved receiver design or increased transmitter power the way you can improve *S/N*. Improving the sensitivity of the receiver does not alter the relative powers of signal and interference. Increasing transmitted power doesn't work either because "one man's signal is another man's interference." That is, if your base station increases transmitter power, then your *S/I* will go up, but users in other cells for whom your base station is an interferer will have their *S/I* go down. If those cells increase transmitter power then you end up where you started because it is the ratio of powers that determines *S/I*.

S/I is primarily a function of the locations of those transmitters that sharing a channel. We will see in a future lecture how to choose an appropriate geometry to achieve adequate *S/I* in a cellular network.

References

1. Rappaport, T. S., *Wireless Communications: Principles and Practice*, Prentice Hall, 1996, ISBN 0-13-375536-3.