Multipath

Introduction

There are two characteristics of radio channels that present serious difficulties for telecommunication systems. One is the tremendous dynamic range that must be accommodated due to the large change in path loss with distance. Another is the presence of multiple propagation paths between transmitter and receiver that produce echoes of the transmitted signal in the received signal. This phenomenon is called *multipath*. Interference between multipath signals can result in *fading*. We'll cover fading in the next lesson.

Theory

Consider an environment like that diagrammed in Figure 7.1 in which there exist objects capable of reflecting radio waves.

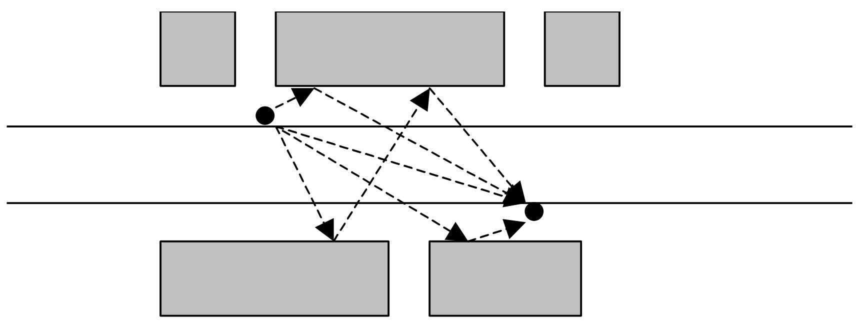

Figure 7.1. In the presence of reflecting objects there can be many paths between two radios, each having a different propagation distance. The situation shown might represent a top view of two radios on either side of a street lined with buildings.

Maxwell's equations are linear. If nothing in the environment is moving or otherwise changing with time then we have a linear, time-invariant system. For such systems the impulse response and transfer function concepts are applicable and generally very useful. Let's take our system input to be the transmitted signal and the output to be the received signal. Conceptually we could transmit an electromagnetic impulse. The impulse would travel along the various paths from transmitter to receiver. The k^{th} path would produce a received impulse after a time delay t_k. Due to the distance traveled and the interaction with reflecting objects, it will have some amplitude a_k. Therefore the total received field, which is the *impulse response* of the transmitter-receiver system will be

$$h(t) = \sum_{k=0}^{N-1} a_k \delta(t - t_k) \tag{7.1}$$

The transfer function or frequency response of the system is the Laplace transform of this evaluated at $s = j\omega = j2\pi f$.

$$
\begin{aligned}
H(\omega) &= \int_0^\infty h(t) e^{-j2\pi f t} dt \\
&= \sum_{k=0}^{N-1} a_k e^{-j2\pi t_k f}
\end{aligned}
\tag{7.2}
$$

This is a sum of phasors. For certain frequencies the phasors might be in phase and add up to a large response. For other frequencies they might add out of phase and give a low response. We see that the effect of mulitpath is to create a response that varies with frequency – a type of frequency selective filter.

Multipath Statistics

Although in principle the impulse response is completely determined by the propagation environment, in practice this environment is so complex that we are inclined to treat the impulse response as a random process. Random processes are typically characterized by their *moments*. The first and second moments are

$$
\begin{aligned}
\langle t \rangle &= \frac{\sum_{k=0}^{N-1} a_k^2 t_k}{\sum_{k=0}^{N-1} a_k^2} \\
\langle t^2 \rangle &= \frac{\sum_{k=0}^{N-1} a_k^2 t_k^2}{\sum_{k=0}^{N-1} a_k^2}
\end{aligned}
\tag{7.3}
$$

(Because the *a*'s represent voltages we square them to get power.) The first moment $\langle t \rangle$ is just the average time of arrival. The first and second moments can be combined to get the *root-mean-square delay spread*:

$$
\sigma_h = \sqrt{\langle t^2 \rangle - \langle t \rangle^2} \tag{7.4}
$$

If we continued to calculate higher-order moments we would get an increasingly accurate description of the impulse response. However, even two moments provides us with useful information, namely, the average time of arrival and the time spread of the multipaths. One function that is completely determined by its first two moments is the Gaussian. So by stopping at the first two moments of $h(t)$ we are effectively modeling it as an equivalent Gaussian with mean $\langle t \rangle$ and variance σ_h^2.

Consider a Gaussian function $a(t)$ with variance σ_h^2, and let's choose a time origin so that the mean is zero. The spectrum of a Gaussian is also Gaussian:

$$a(t) = \frac{1}{\sqrt{2\pi}\sigma_h} e^{-\frac{1}{2}\left(\frac{t}{\sigma_h}\right)^2} \Leftrightarrow A(f) = e^{-2\pi^2\sigma_h{}^2 f^2} \tag{7.5}$$

The spectrum is down to ½ its peak value when $e^{-2\pi^2\sigma_h{}^2 f^2} = 1/2$. Solving for the frequency we find $f \approx 1/5\sigma_h$. We can take this as the bandwidth of the Gaussian. We will use this to define the *coherence bandwidth* of the impulse response:

$$B_c = \frac{1}{5\sigma_h} \tag{7.6}$$

The coherence bandwidth specifies the minimum frequency deviation over which the transfer function can change significantly. In other words, if two signals differ in frequency by much less than B_c then they experience essentially the same response through the physical channel. If two signals differ by much more than B_c then they may experience completely different responses.

The relation between the signal bandwidth B and the coherence bandwidth of the physical channel B_c is important. If $B << B_c$ then all frequency components in the signal are essentially modified in the same manner, and the received signal is simply a time-delayed and attenuated version of the transmitted signal. In effect the channel is simply an attenuator. This is referred to as *flat fading*. On the other hand if $B_c << B$ then different frequency components are modified differently by the channel, and the received signal can be seriously distorted. In effect the channel is then a filter with some complicated transfer function. This is referred to as *frequency selective fading*. In this case we often must take steps to undo the distortion introduced by the physical channel. Some ways to do this are through the use of *equalizers*, or, in CDMA systems, *rake filters*. But, more about that later.

Example 7.1

Assume $h(t) = \sum_{k=0}^{N-1} a_k \delta(t - t_k)$ with $a_0 = 1, t_0 = 0\mu s$, $a_1 = 0.5, t_0 = 1\mu s$, $a_2 = 0.25, t_2 = 1.5\mu s$. Then

$$\langle t \rangle = \frac{(1)^2(0) + (0.5)^2(1) + (0.25)^2(1.5)}{(1)^2 + (0.5)^2 + (0.25)^2} \mu s = 0.262\mu s$$

$$\langle t^2 \rangle = \frac{(1)^2(0)^2 + (0.5)^2(1)^2 + (0.25)^2(1.5)^2}{(1)^2 + (0.5)^2 + (0.25)^2} \mu s^2 = 0.298\mu s^2$$

so $\sigma_h = \sqrt{0.298 - 0.262^2} = 0.479\mu s$ and $B_c = 0.418MHz$.

Simulation

Figure 7.2 shows a Mathcad program that simulates a channel in which there is a direct path and four other multipaths with random times of arrival spread over about 100 ns and random amplitudes. You can see from the plot that the frequency response varies quite erratically over the 100 MHz bandwidth shown. Different radio channels in this bandwidth might experience as much as a 30-dB difference in fading response. The coherence bandwidth comes out at a little more than 10 MHz. You can see from the plot that this is roughly the characteristic width of the bumps in the frequency response.

number of multipaths $N := 4 \qquad k := 0..N$

maximum delay time $t_{max} := 10^{-7}$

random time delays and amplitudes $t_k := rnd(t_{max}) \qquad a_k := rnd(1)$

$t_0 := 0 \qquad a_0 := 1$

frequency response $H(f) := \sum_k a_k \cdot e^{-j \cdot 2 \cdot \pi \cdot t_k \cdot f}$

$$t_1 := \frac{\sum_k (a_k)^2 \cdot t_k}{\sum_k (a_k)^2} \qquad t_2 := \frac{\sum_k (a_k)^2 \cdot (t_k)^2}{\sum_k (a_k)^2}$$

$$\sigma := \sqrt{t_2 - t_1^{\,2}} \qquad B := \frac{1}{5 \cdot \sigma} \qquad \frac{B}{10^6} = 11.653$$

$$f := 1800 \cdot 10^6, 1801 \cdot 10^6 .. 1900 \cdot 10^6$$

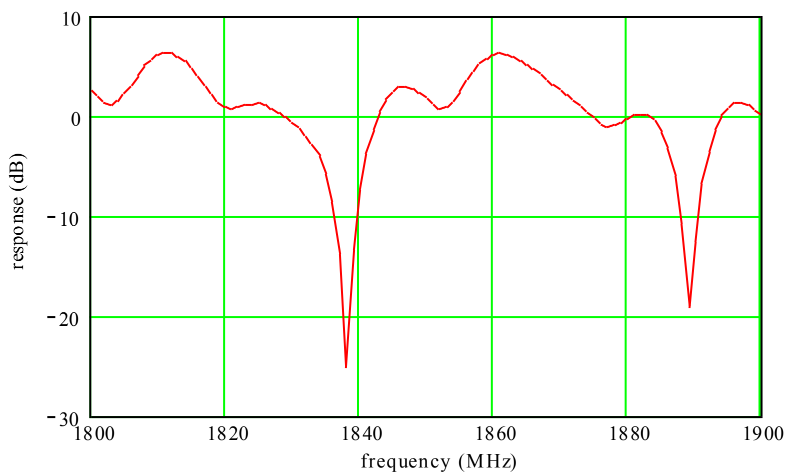

Figure 7.2. Simulation of frequency response in a multipath environment. A direct path and four multipaths with random

amplitudes and time delays between 0 and 0.1μs interfere to create this response.

Experiment

A network analyzer was used to measure $h(t)$ and $H(\omega)$ in a laboratory in the EME building at WSU. The frequency band was 700 to 1000 MHz resulting in 1-m resolution. Figures 7.3-5 show some results.

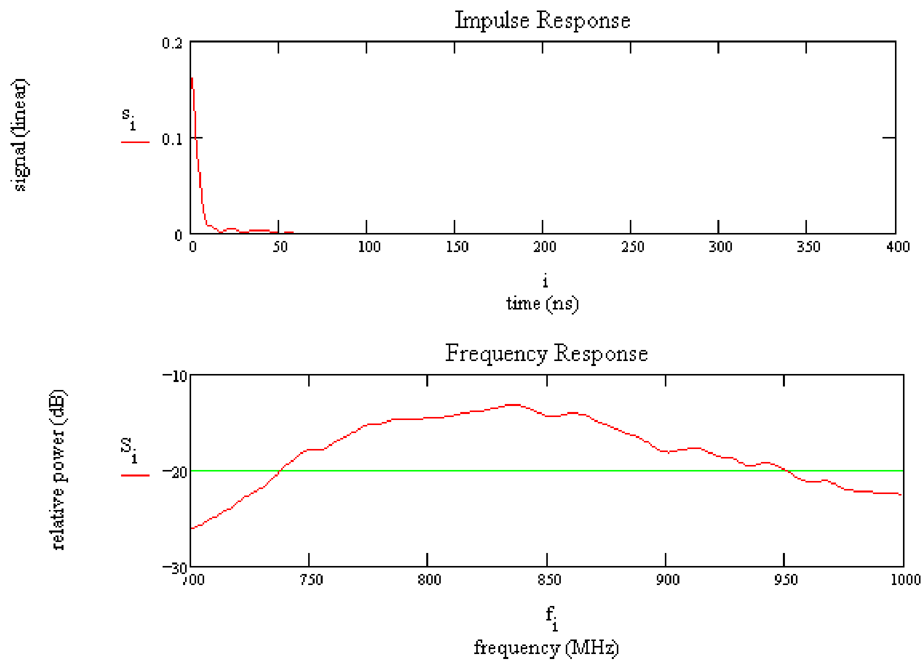

Figure 7.3: Impulse and frequency responses when antennas are next to each other. The roll-off in frequency response is primarily due to antenna resonance.

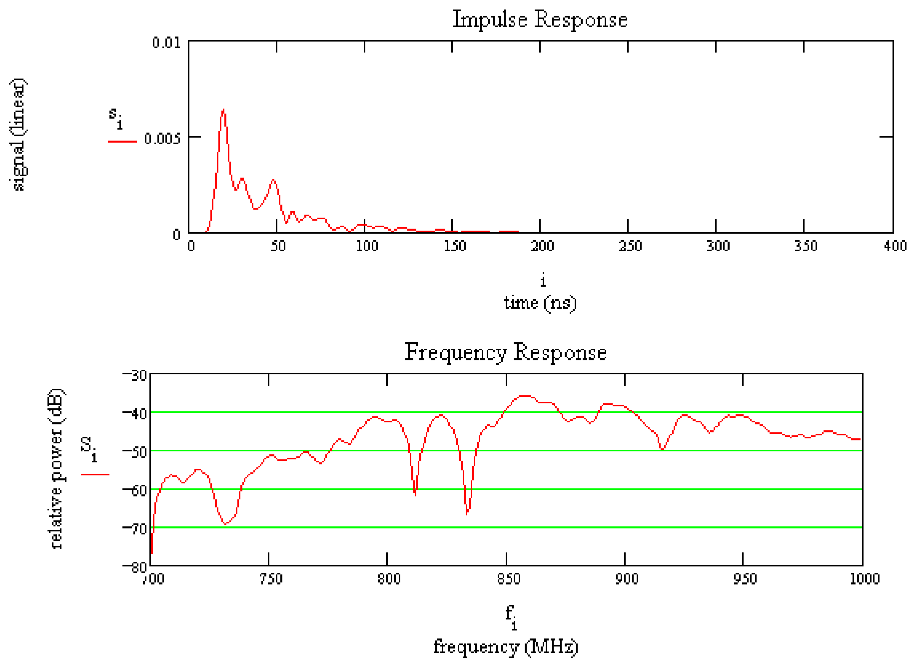

Figure 7.4: Responses for antennas separated by about 20 ft and having a clear line-of-sight path between them.

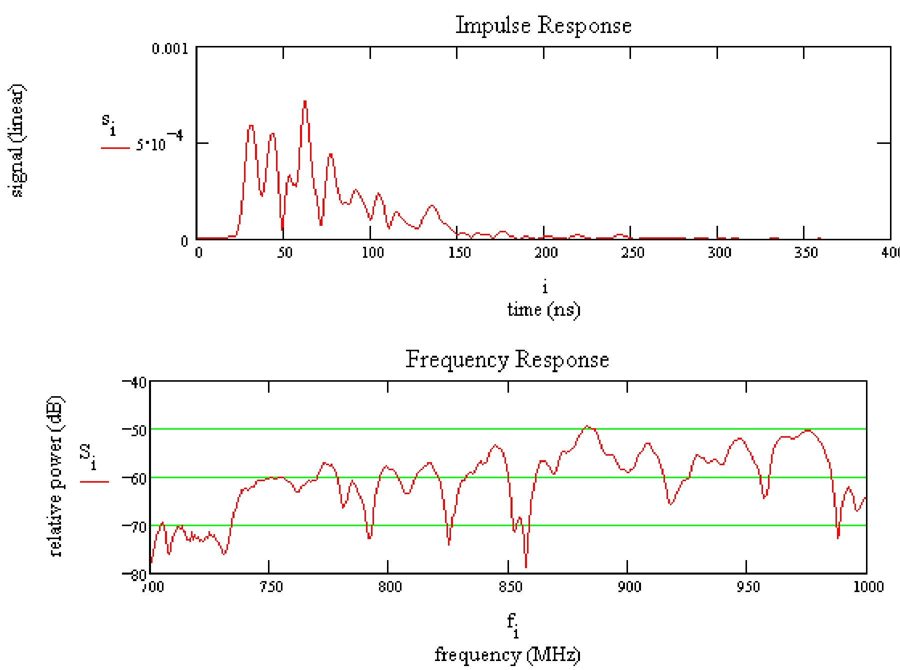

Figure 7.5: Responses for antennas separated by about 30 ft and no line-of-sight path between them.

From the above measurements we can see that, even with the transmitter and receiver in the same room, we can get a very complicated frequency response. If you measured the frequency response of a coaxial cable and saw something like this, you'd get a new cable. But in a wireless environment this is the type of physical channel we are stuck with. In fact, it gets worse, because as soon as the receiver starts to move, e.g., a mobile phone, this ugly frequency response can rapidly change in an essentially random manner. Our wireless communication systems must, therefore, be designed to function under such non-ideal circumstances.

References

1. Rappaport, T. S., *Wireless Communications: Principles and Practice*, Prentice Hall, 1996, ISBN 0-13-375536-3.