Lecture 6

Link Budgets

1 Introduction

One of the most important uses of propagation models is to develop *link budgets*. A financial budget typically involves looking at your income, looking at your required expenditures, and making sure income is at least as large as expenditures. A link budget is a similar process in an RF system involving power instead of money.

2 Probability Received Signal Exceeds Some Value

One characterization of a radio receiver is its sensitivity. This is the smallest received signal that it can properly operate with. For example, suppose a certain cellular phone must receive at least $P_{min}=-100\text{ dBm}$ in order to function properly. Say received signal strength is modeled by

$$P_R=P_0-10n\log\frac{r}{r_0}+X \tag{1}$$

and the phone is some distance r from the base station. An important question is: What is the probability that P_R will be at least P_{min}? This is the probability that the phone will function properly. P_R is a Gaussian random variable with mean $P_{R0}=P_0-10n\log\frac{r}{r_0}$ and variance σ^2. The probability distribution function for P_R will look like that shown in XXXX, and the desired probability is the area under the curve from P_{min} to infinity.

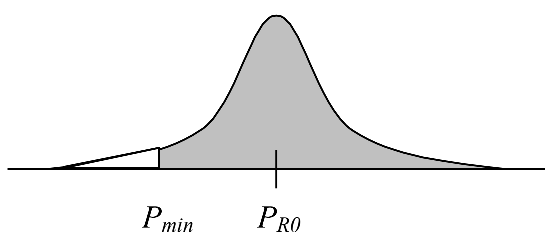

Fig. 1 Probability that P_R is at least P_{min} is the area of the shaded portion of the bell curve.

Mathematically this is

$$\Pr[P_R\geq P_{min}]=\frac{1}{\sigma\sqrt{2\pi}}\int_{P_{min}}^{\infty} e^{-\frac{1}{2}\left(\frac{x-P_{R0}}{\sigma}\right)^2}dx \tag{2}$$

With a change of variable $y=(x-P_{R0})/\sigma$ this becomes

$$\Pr[P_R \geq P_{min}] = \frac{1}{\sqrt{2\pi}} \int_{\frac{P_{min}-P_{R0}}{\sigma}}^{\infty} e^{-\frac{1}{2}y^2} dy$$
$$= Q\left(\frac{P_{min}-P_{R0}}{\sigma}\right) \quad (3)$$

where the Q function is defined as

$$Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^{\infty} e^{-\frac{1}{2}y^2} dy \quad (4)$$

The *error function* erf(z) is related to the Q function by

$$Q(z) = \frac{1}{2}\left[1 - \mathrm{erf}\left(\frac{z}{\sqrt{2}}\right)\right] \quad (5)$$

Scilab and Matlab have built-in versions of erf. When $z \geq 0$ the following approximation for Q

$$Q(z) = \frac{1}{0.661\,z + 0.339\sqrt{z^2+5.51}} \frac{e^{-\frac{1}{2}z^2}}{\sqrt{2\pi}} \quad (6)$$

is good to better than 0.3%. For $z<0$ we can use $Q(z)=1-Q(-z)$.

Example 1: For the typical car, a certain FM radio station currently produces a signal $P_R = -40 - 35\log(r/1\,\mathrm{km}) + X$ with $\sigma = 8\,\mathrm{dB}$. What is the probability of at least $P_{min} = -100\,\mathrm{dBm}$ at a distance of 100 km?

At 100 km, $P_{R0} = -40 - 35\log(100) = -110\,\mathrm{dBm}$. Since $Q\left(\frac{-100-(-110)}{8}\right) = 0.106$ the probability is about 10.6%.

Often we want to go the other way with such a calculation, inverting the Q function to find out the transmitted power level required for good reception.

Example 2: For the situation the previous example, how much would we have to increase the transmitted power to achieve 90% probability of good reception?

We need to find z such that $Q(z) = 0.9$. Here is how we can do that in Scilab

```
-->deff('y=Q(z)','y=0.5*(1-erf(z/sqrt(2)))');

-->deff('y=f(z)','y=Q(z)-0.9');

-->fsolve(0,f)
 ans  =  - 1.2815516
```

So $z = -1.282$. We want to have

$$\left(\frac{-100 - P_{R0}}{8}\right) = -1.282$$

Solving we get $P_{R0} = -90\,\text{dBm}$. This is 20 dB larger than the previous value of $-110\,\text{dBm}$. So we need to increase transmitted power by 20 dB (a factor of 100).

If we can't increase transmitted power, we will have to live with a smaller coverage area.

Example 3: Suppose we have to live with the original power level. At what distance will the probability of good reception be 90%?

We still need $P_{R0} = -90\,\text{dBm}$ but we achieve this by reducing the distance *r* so that

$$-40 - 35\log(r/1\,\text{km}) = -90$$

The solution is $r = 26.8\,\text{km}$.

3 Fractional Coverage

A related problem is as follows. We have a base station that is responsible for providing a signal to all users within a distance *R* of the station. Given a log-normal shadowing model and a value of P_{min} , what fraction of users will be able to communicate with the base station? This is the same as the average probability of getting a good signal, the averaging being done over the area of the radio cell. Computing this average gives us

$$\begin{aligned} U(P_{min}) &= \frac{\iint \Pr[P_R > P_{min}]\,dA}{\iint dA} \\ &= \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \Pr[P_R > P_{min}]\,r\,dr\,d\theta \\ &= \frac{2}{R^2}\int_0^R Q\left(\frac{P_{min} - (P_0 - 10\,n\log r/r_0)}{\sigma}\right) r\,dr \end{aligned} \tag{7}$$

With a fair amount of manipulation this can be put into the form

$$U(P_{min}) = \frac{1}{2}[1 - \text{erf}(a)] + \frac{1}{2} e^{\frac{1-2ab}{b^2}}\left[1 + \text{erf}\left(a - \frac{1}{b}\right)\right] \tag{8}$$

with

$$\begin{aligned} a &= \frac{P_{min} - \left(P_0 - 10\,n\log\dfrac{R}{r_0}\right)}{\sqrt{2}\,\sigma} \\ b &= \frac{10\,n}{\sqrt{2}\,\sigma\ln 10} \end{aligned} \tag{9}$$

A common use of link budget theory is to calculate the transmitted power needed to provide a given percentage coverage in a cell of a certain size, or the maximum cell size possible with a given transmitted power.

Example 4: Assume that 1900-MHz cell phones transmit 800 mW and have 0-dBi gain antennas. Your base station has a 6-dBi antenna and needs at least $-110\,\text{dBm}$ of signal. Assume that fields propagate as in free space out to 10 m and then experience log-normal shadowing with $n=3$ and $\sigma=10\,\text{dB}$. How large a cell radius can these phones operate within with 95% coverage area?

From (9) we have $b=0.9212$. In Scilab we define

```
deff('y=U(a,b)','y=0.5*(1-erf(a))+0.5*exp((1-2*a*b)/
(b^2))*(1+erf(a-1/b))');
```

With $b=0.9212$, trial and error with the *a* value leads us to

```
-->U(-0.8570,0.9212)
 ans  =  0.9500116
```

so $a=-0.8570$. The 800 mW of transmitted power is 29 dBm. The wavelength is $300/1900=0.158\,\text{m}$. The fields propagate as in free space out to 10 m, so

$$P_0=29+6+0-20\log\frac{4\pi(10\,\text{m})}{0.158\,\text{m}}=-23\,\text{dBm}$$

at $r=r_0=10\,\text{m}$. From larger values of *r*

$$P_{R0}=-23-30\log\frac{r}{10\,\text{m}}$$

We require

$$-0.857=\frac{-110-\left(-23-30\log\frac{R}{10\,\text{m}}\right)}{\sqrt{2}(10)}$$

The solution is $R=3.13\,\text{km}$.

4 References

1. Rappaport, T. S., *Wireless Communications: Principles and Practice*, Prentice Hall, 1996, ISBN 0-13-375536-3.