Lecture 4

Empirical Propagation Models

1 Introduction

We need to be able to predict received signal strength in wireless systems. We use these predictions to figure things out such as how much area a certain base station can cover, or how far away from a base station a cell phone can be and still communicate, or how much interference our channel will get from other phone uses. Factors like transmitted power and antenna gains are easy to account for, but the effects of propagation are generally not. We have seen that even in the highly idealized case of two antennas above a perfectly flat ground, the dependence of received power on distance is very different than in free space.

In principle, we could take a precise description of the terrain, buildings, foliage and so on in a given area and solve Maxwell's equations in this environment to arrive at an exact prediction of the field at all points of interest. Numerical techniques, such as the finite-difference time-domain (FDTD) method, can accomplish this with fine accuracy even in very complex environments. There are, however, two problems with such an approach. First it requires a detailed description of the environment that typically just isn't available and would be very hard to develop. Second, it is (currently) computationally feasible only for relatively small volumes.

Instead, we will typically use rather simplified *empirical propagation models* with a few parameters that can be adjusted to give reasonable predictions. Empirical refers to the fact that these models are generally derived by making lots of measurements and then fitting physically motivated functions to the observations. When more accurate predictions are required we can "split the difference" with an exact solution by adding some information about the environment.

2 Log-Normal Shadowing

2.1 Theory

The most important aspect of propagation for us is that the field strength generally decreases with distance from the transmitter. You might think this is a bad thing but we'll see that it actually is very useful if you want to build a system that serves many users. We have already seen examples where $P_r = P_0 - 10n\log(r/r_0)$. (Note that we'll drop the dBm and dB subscripts when writing power and gain, as we will almost always be working on a log scale from here on out.) Free space propagation is the case $n=2$. The two-ray case above a ground plane with $\Gamma=-1$ has $n=4$ (at large distances from the transmitter). We should expect the real world to be more complicated than either of these two idealized cases, of course, but they might, hopefully, still capture much of the general behavior of the field in certain real situations. Let's generalize things a bit and write

$$P_r = P_0 - 10n\log\frac{r}{r_0} + X \tag{1}$$

where n is free to take on any value, and X is a random variable that represents the failure of the model to perfectly represent the real world. We'll assume that X is zero-mean Gaussian, that is, that it has a probably distribution

$$p_X(X)=\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{1}{2}\left(\frac{X}{\sigma}\right)^2} \tag{2}$$

A model of this form is usually referred to as *log-normal shadowing* because the log of the power has a normal (Gaussian) distribution which typically represents the effects of shadowing by obstacles. You can think of □ as accounting for our ignorance. If σ is small, then our model gives very accurate predictions. On the other hand, if σ is large then the model gives poor predictions. In the wireless literature σ is typically assumed to be around 8 dB.

There are two parameters, P_0 and n, in model (1). How do we know what these are? Most commonly many measurements are made and then a *linear regression* is performed to find the parameters that give the "best fit," i.e., the smallest σ. We'll see how to do a linear regression shortly.

2.2 Simulation

Fig. 1 shows a log-normal shadowing simulation. In the left frame there are no random fluctuations in P_r and we obtain circular power contours. In the right frame random fluctuations (the variable X) result in more complicated power contours.

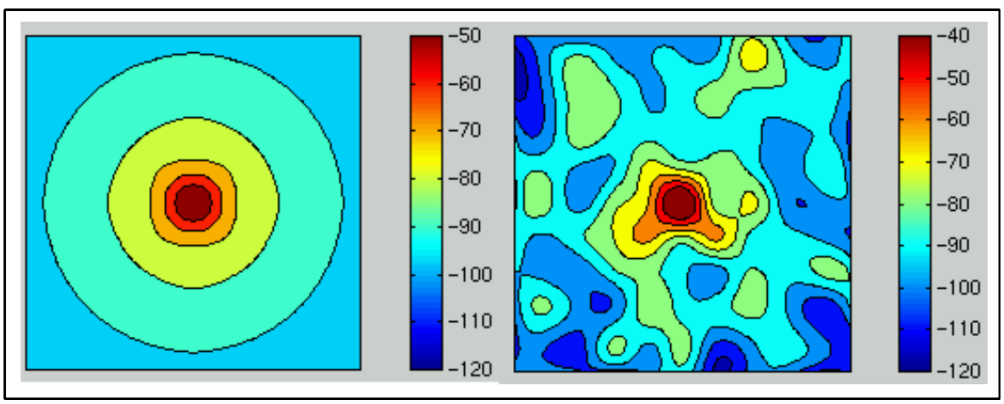

Fig. 1 Log-normal shadowing simulation. Received power is plotted vs. ground position. Color bar shows signal level in dBm. Model was $P_r=P_0-10\,n\log(r/r_0)+X$ *with* $P_0=-40\,\text{dBm}$, $r_0=100\,\text{m}$ *and* $n=4$. *Simulated region is 4-km on a side. Left frame has* $\sigma=0$, *right frame has* $\sigma=8\,\text{dB}$.

Although this is simulated data, looking at the contours in the $\sigma=8\,\text{dB}$ case you might be able to visualize how obstructions (hills, etc.) could create the shadow regions corresponding to low power levels.

2.3 Experiment

Fig. 2 shows the probability distributions of the residuals of fits of the model (1) for two cellular control channels in the Pullman, WA area, i.e., the random variable *X*. The distributions are not exactly Gaussian, but the Gaussian model gives a workable approximation.

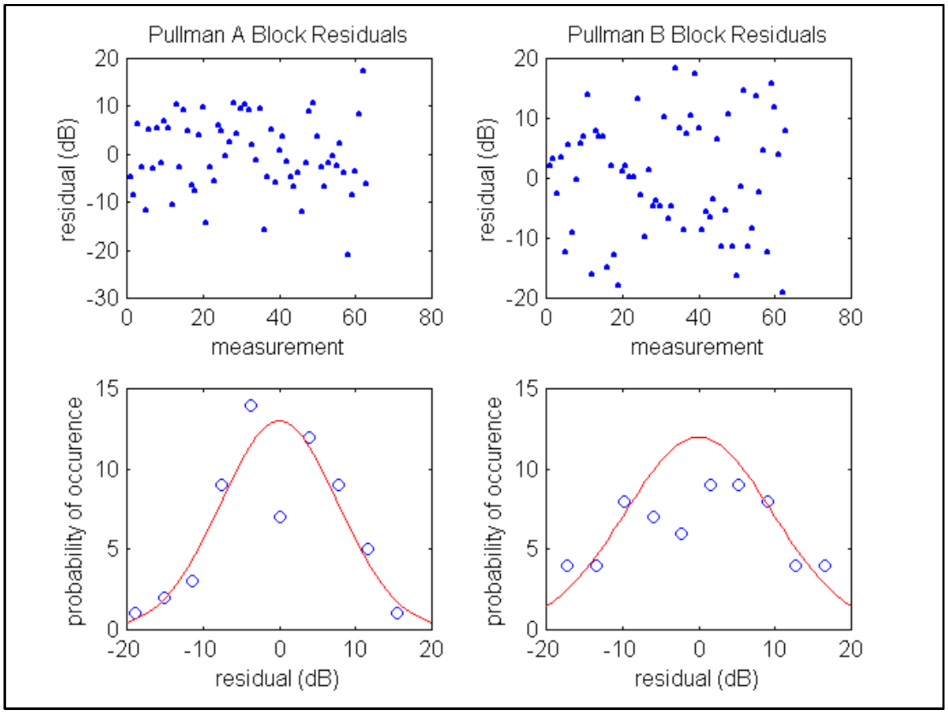

Fig. 2 Residuals (random variable "X") resulting from using model (1) to fit cellular control channel signals in the Pullman, WA area. The solid curves are best-fit Gaussian distributions.

3 Linear Regression

Many physical processes are well modeled by linear ($y=a+bx$), power-law ($y=a\,x^b$), or exponential ($y=a\,e^{-bx}$) relationships. For a power-law we can take the logarithm of both sides to get $\log y=\log a+b\log x$ which shows that the relation between $\log x$ and $\log y$ is linear. For exponential relationships we can write $\ln y=\ln a-bx$ which is a linear relationship between x and $\ln y$. So all three of these types of models can be represented by a linear relationship. It should come as no surprise, therefore, that being able to fit a line to data is an important skill for scientists and engineers. Let's see how this is done.

Let's say you have some data points (x_i, y_i) , for $i=1,2,\ldots,N$, and you want to fit the linear model $y=a+bx$. Let's take the differences between the actual *y* values (y_i) and the modeled *y* values ($a+b\,x_i$), square them, add them up, and divide by the total number of points. This gives us the *mean square error*:

$$MSE=\frac{1}{N}\sum_{i=1}^{N}\left[y_i-(a+b\,x_i)\right]^2 \tag{3}$$

We want the *MSE* to be as small as possible. We can set $\frac{\partial MSE}{\partial a}=\frac{\partial MSE}{\partial b}=0$ to find the values of a and b that do this. We arrive at

$$\begin{aligned} b&=\frac{\langle xy\rangle-\langle x\rangle\langle y\rangle}{\langle x^2\rangle-\langle x\rangle^2} \\ a&=\langle y\rangle-b\langle x\rangle \end{aligned} \tag{4}$$

where the brackets denote the mean value of the quantity. For example $\langle x^2\rangle=\frac{1}{N}\sum_{i=1}^{N} x_i^2$. Using these values we can estimate the variance of X by

$$\sigma^2=\frac{1}{N-2}\sum_{i=1}^{N}\left[y_i-(a+b\,x_i)\right]^2 \tag{5}$$

This is $N/(N-2)$ times the *MSE*. The –2 in the denominator is due to the factor that our model has two free parameters, a and b. This results in the *MSE* generally being smaller than the variance of X because the model is able to fit not only the "true" line but also some of X.

Example 1: Suppose we measure received signal strength at distances of 100, 200, 1000, and 3000 meters and obtain values 0, –20, –35, and –70 dBm, respectively. We want to fit a model of the form (1). The first step is to convert distances (r) to a log scale to serve as x values. Let's take $r_0=100\,\mathrm{m}$ and use Scilab to perform the calculations.

```
-->r = [100,200,1000,3000];
-->x = log10(r/100)
 x  =  0.    0.30103    1.    1.4771213
-->y = [0,-20,-35,-70];
-->b = (mean(x.*y)-mean(x)*mean(y))/(mean(x.^2)-mean(x)^2)
 b  =  - 42.891233
-->a = mean(y)-b*mean(x)
 a  =  - 1.4604167
```

Our model is therefore $P_r=-1.5-43\log\frac{r}{100\,\mathrm{m}}+X$. To estimate the standard deviation of X we calculate

```
-->yf = a+b*x
 yf  =  - 1.4604167  - 14.371964  - 44.35165  - 64.815969
-->sigma = sqrt(sum((yf-y).^2)/(4-2))
 sigma  =  8.6062517
```

so $\sigma=8.6\,\mathrm{dB}$.

4 Reciprocity

It is important to keep in mind that most of the radio links we are interested in are two-way, or *full-duplex* links. This means that both radios act as transmitter and receiver. This is diagrammed in the following figure.

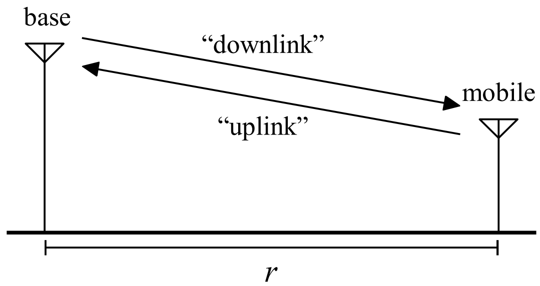

Fig. 3: In a full duplex radio link, both radios act as transmitter and receiver to achieve two-way communication.

In a cellular system, typically one of the radios is a fixed *base station* and the other is a *mobile* handset or cell phone. We call the link in which the base transmits and the mobile receives the *downlink* (also called the forward channel) and that in which the mobile transmits and the base receives the *uplink* (also called the reverse channel). Let's say the base transmits power P_{TB} and the mobile receives P_{RM}. If the mobile then transmits power P_{TM} what will the base receive?

The *reciprocity theorem* of electromagnetics provides the answer. If each radio uses the same antenna for both transmit and receive, and if the environment in which the fields propagate contain only "non-exotic" materials then can write

$$\begin{aligned} P_{RM} &= P_{TB} - L \\ P_{RB} &= P_{TM} - L \end{aligned} \tag{6}$$

where all quantities are on a dB scale. (Note that there are exotic "non-reciprocal" materials, notably ferrite, but we do not encounter them in the scenarios we are considering.) Here L is the total system loss between transmitter and receiver. The important point is that loss does not depend on which radio acts as transmitter and which acts as receiver. Therefore if we develop a propagation model for the downlink, it will also apply to the uplink. We need only take into account the difference in transmitted power.

5 References

1. Rappaport, T. S., *Wireless Communications: Principles and Practice*, Prentice Hall, 1996, ISBN 0-13-375536-3.
2. Balanis, C. A., *Advanced Engineering Electromagnetics*, Wiley, 1989, ISBN 0-471-62194-3.