Lecture 3

Ground Reflections

1 Introduction

When dealing with radio waves in a terrestrial environment we almost never observe free-space propagation of the type we talked about in the last lecture. This is because the waves interact with the environment: ground, trees, buildings, etc. The result is that we typically have several reflected or diffracted waves interfering with one another at any given point. (We'll talk about diffraction later.) This complicates things quite a bit. In this lesson we'll consider the simple model of a transmitter-receiver pair above an infinite "ground plane." We don't live on an infinite flat plane, but this simple model does have some explanatory power in the real world, and it will motivate some empirical models that we'll learn about later.

2 Geometry

Assume a transmitter is a height h_t above a flat, smooth ground and a receiver is at a height h_r while the ground distance between them is *r*. This is illustrated in Fig. 1. Let the reflection coefficient of the ground be Γ. The reflection coefficient is the ratio of the reflected electric field to the incident electric field. In our model we will assume this is a constant. In fact, as you know from a course like EE 351, this is really a function of the field polarization and the angle of incidence.

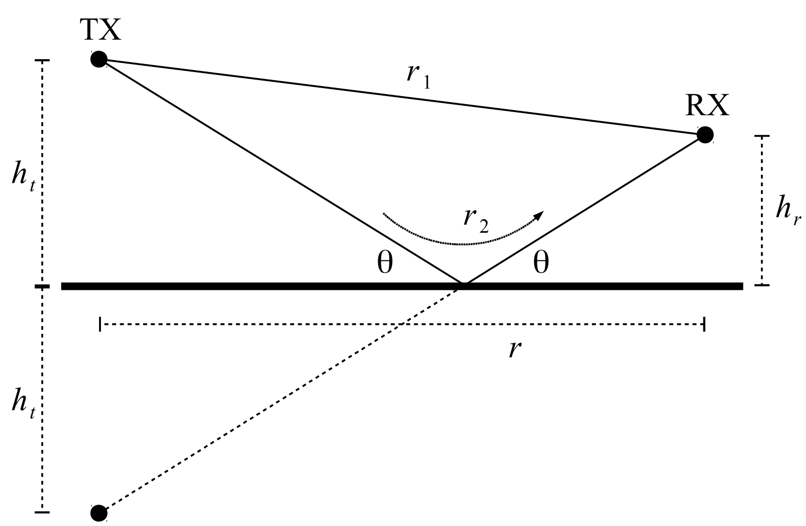

Fig. 1: Geometry for the ground reflection problem (the "two-ray model").

There are two paths for radio waves to take from transmitter to receiver: a direct path and a reflected path. Call the length of the direct path r_1 and that of the reflected path r_2. The law of reflection requires the angle of incidence to equal the angle of reflection (the angles θ in the figure). The result is that the reflected field appears to come from a "mirror image" of the source at a distance h_t below the ground. Basic geometry gives us the distances

$$r_1=\sqrt{r^2+(h_t-h_r)^2}$$
$$r_2=\sqrt{r^2+(h_t+h_r)^2} \quad (1)$$

We'll be interested in cases where $r \gg h_t, h_r$. For example r might be hundreds or thousands of meters while the h values are only a few, or maybe a few tens of meters. Then the following approximations are good (recall that for small x, $\sqrt{1+x} \approx 1+x/2$):

$$r_1 \approx r+\frac{(h_t-h_r)^2}{2r}$$
$$r_2 \approx r+\frac{(h_t+h_r)^2}{2r} \quad (2)$$

The path difference is

$$\Delta r=r_2-r_1 \approx 2\frac{h_t h_r}{r} \quad (3)$$

As we'd expect from Fig. 1, this goes to zero at large distances.

Exercise 1: Derive (2) and (3) using the approximation $\sqrt{1+x} \approx 1+x/2$.

3 Theory

The receiver will "see" two transmitters – the real transmitter above the ground and a virtual, or mirror image, transmitter below the ground. The intensity of each of these fields in the absence of the other would be given by the Friis equation:

$$P_{R1}=P_T\frac{G_T G_R \lambda^2}{(4\pi)^2}\,\frac{1}{r_1^2} \text{ and } P_{R2}=P_T\frac{G_T G_R \lambda^2}{(4\pi)^2}\,\frac{1}{r_2^2} \quad (4)$$

When both are present we cannot simply add the powers because it is the EM fields that add, and the fields have both amplitude and phase. The amplitude is proportional to the square root of the intensity and the phase is 2π times the distance traveled in wavelengths. The amplitude of the reflected field is also multiplied by Γ, the reflection coefficient of the ground. The total power, proportional to the magnitude squared of this total field, is

$$P_R=P_T\frac{G_T G_R \lambda^2}{(4\pi)^2}\left|\frac{e^{-j\frac{2\pi}{\lambda}r_1}}{r_1}+\Gamma\frac{e^{-j\frac{2\pi}{\lambda}r_2}}{r_2}\right|^2 \quad (5)$$

For large r (2) tells us that r_1, r_2 both approach r, so the phase terms in (5) will approach one another in both phase and amplitude (other than the Γ factor). If $\Gamma=-1$, which would correspond to a perfectly conducting ground with a certain polarization, the two terms will tend to cancel each other and the received power will drop very rapidly. We have in this special case

$$
\begin{aligned}
\left| \frac{e^{-j\frac{2\pi}{\lambda}r_1}}{r_1} + \Gamma \frac{e^{-j\frac{2\pi}{\lambda}r_2}}{r_2} \right|^2 &\approx \frac{1}{r^2}\left| e^{-j\frac{2\pi}{\lambda}r_1} - e^{-j\frac{2\pi}{\lambda}r_2} \right|^2 \\
&= \frac{1}{r^2}\left| 1 - e^{-j\frac{2\pi}{\lambda}\Delta r} \right|^2 \\
&= \frac{1}{r^2}\left| e^{j\frac{\pi}{\lambda}\Delta r} - e^{-j\frac{\pi}{\lambda}\Delta r} \right|^2 \\
&= \frac{4}{r^2}\sin^2\frac{\pi\Delta r}{\lambda}
\end{aligned} \tag{6}
$$

so

$$
P_R \approx P_T \frac{G_T G_R \lambda^2}{(4\pi)^2} \cdot \frac{4}{r^2}\sin^2\frac{\pi\Delta r}{\lambda} = P_T \frac{G_T G_R \lambda^2}{(2\pi r)^2}\ \sin^2\frac{\pi\Delta r}{\lambda} \tag{7}
$$

Using (3)

$$
P_R \approx P_T \frac{G_T G_R \lambda^2}{(2\pi r)^2}\ \sin^2\frac{2\pi h_t h_r}{\lambda r} \tag{8}
$$

The sine term oscillates between 0 and 1 while the "envelope" $P_T G_T G_R \lambda^2/(2\pi r)^2$ has the $1/r^2$ behavior of free-space propagation.

The approximation $\sin^2(1/x) = 1/x^2$ is good to 1.5 dB for $x \ge 1$. (That is, $10\log\left[(1/x^2)/\sin^2(1/x)\right] < 1.5\,\mathrm{dB}$ for $x \ge 1$.) So, for large enough *r,* we can write

$$
P_R \approx P_T G_T G_R \frac{h_t^2 h_r^2}{r^4} \tag{9}
$$

Here the received power is falling off as $1/r^4$, much more rapidly than it would in free space. It is also interesting to note that there is no wavelength dependence in this expression. The envelopes of (8) and (9) are equal when $r = 2\pi h_t h_r/\lambda$. If we neglect the sine oscillations in (8) we can create a composite propagation model as follows:

$$
P_R = \begin{cases} P_T G_T G_R \dfrac{\lambda^2}{(2\pi r)^2} & ;\ r < \dfrac{2\pi h_t h_r}{\lambda} \\[2ex] P_T G_T G_R \dfrac{h_t^2 h_r^2}{r^4} & ;\ r \ge \dfrac{2\pi h_t h_r}{\lambda} \end{cases} \tag{10}
$$

This is an example of a *breakpoint* model, the breakpoint $r = 2\pi h_t h_r/\lambda$ being where the model changes from one type of behavior to another. If we use the breakpoint as the reference distance r_0 then

$$P_{R,dBm}=\begin{cases} P_{0,dBm}-20\log\frac{r}{r_0} & ;\ r<r_0 \\ P_{0,dBm}-40\log\frac{r}{r_0} & ;\ r\ge r_0 \end{cases} \tag{11}$$

$$r_0=2\pi h_t h_r/\lambda$$
$$P_{0,dBm}=P_{T,dBm}+G_{T,dB}+G_{R,dB}+20\log\frac{\lambda^2}{(2\pi)^2 h_t h_r}$$

(Keep in mind that everything we've done is valid only for the case $\Gamma=-1$.) The important point here is that the presence of the ground can cause the field to decay very differently from the $1/r^2$ behavior of free space.

It is instructive to ask what happens to (11) if we change the height of one or both of the antennas. First, notice that the model depends on ly on the product $h_t h_r$. Let's say we change the heights so that the new value of this product is $a h_t h_r$. For example, if we double the height of one of the antennas then $a=2$. The new breakpoint will be $r_0'=a r_0$. Since there is no change in the transmitted power or antenna gains the new reference power is

$$\begin{aligned} P'_{0,dBm}&=P_{T,dBm}+G_{T,dB}+G_{R,dB}+20\log\frac{\lambda^2}{(2\pi)^2 a h_t h_r} \\ &=P_{0,dBm}-20\log a \end{aligned} \tag{12}$$

(because $20\log\frac{x}{a}=20\log x-20\log a$). If we are at a small distance where $r<r_0, r_0'$ then the new model is

$$\begin{aligned} P'_{R,dBm}&=P'_{0,dBm}-20\log\frac{r}{a r_0} \\ &=P_{0,dBm}-20\log a-20\log\frac{r}{r_0}+20\log a \\ &=P_{R,dBm} \end{aligned} \tag{13}$$

So, for distances less that either breakpoint there is no change in the received power. This makes sense because () models the close-in field as being the same as in free-space. For small distances the ground is assumed to have no effect, hence the antenna heights shouldn't have any effect.

On the other hand, at larger distances where $r>r_0, r_0'$

$$\begin{aligned} P'_{R,dBm}&=P'_{0,dBm}-40\log\frac{r}{a r_0} \\ &=P_{0,dBm}-20\log a-40\log\frac{r}{r_0}+40\log a \\ &=P_{R,dBm}+20\log a \end{aligned} \tag{14}$$

and received power is changed by the constant amount $20 \log a$. For example, if you increase either antenna height by a factor of 2, then the received power at large-enough distances increases by 6 dB.

The following figure shows an example in which $h_t h_r$ is increased by a factor of 10.

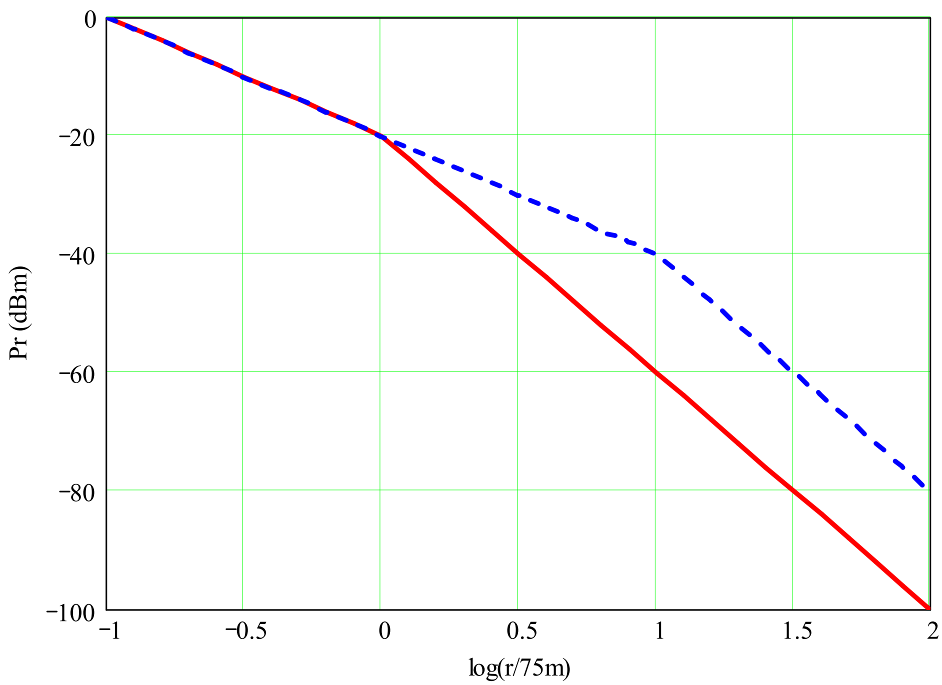

Fig. 2 Effect of changing antenna height. Solid curve shows P_R *for* $h_t = h_r = 2\,\mathrm{m}$, , $G_R = 3\,\mathrm{dB}$, $G_T = 10\,\mathrm{dB}$ *and* $P_T = 1\,\mathrm{W}\,(30\,\mathrm{dBm})$ *. Dashed curve is the case where we change to* $h_t = 20\,\mathrm{m}$ *. The effect is to extend the breakpoint to a larger distance.*

4 Simulation

Fig. 3 shows a calculation of (5) for the case $\Gamma = -1$ and $h_t = 10\lambda, h_r = 3\lambda$. Also shown are $1/r^2$ and $1/r^4$ behaviors characteristic of small and large *r* values.

When received power falls off as $1/r^n$, we say that the *propagation constant* is *n*. For free space the propagation constant is $n = 2$. At large distances the ground-reflection model with $\Gamma = -1$ has a propagation constant $n = 4$. Thus we see that it is quite possible, even in a relatively simple geometry, to get a propagation constant different than that of free space.

Example 1: Assume a propagation model of the form (11). Suppose a cellular base station transmits 30 dBm (1 W) of power and has antenna gain 7 dB and height 10 m. A roof-mounted car antenna is 1.5-m high and has 3 dB gain. The frequency is 1900 MHz. How far away can the car be and still receive $-90\,\mathrm{dBm}$ of signal?

The wavelength is $300/1900 = 0.158\,\mathrm{m}$. From (11) we get $r_0 = 597\,\mathrm{m}$ and $P_0 = -47.5\,\mathrm{dBm}$. When then solve $-90 = -47.5 - 40 \log(r/597\,\mathrm{m})$ to get $r = 6.89\,\mathrm{km}$.

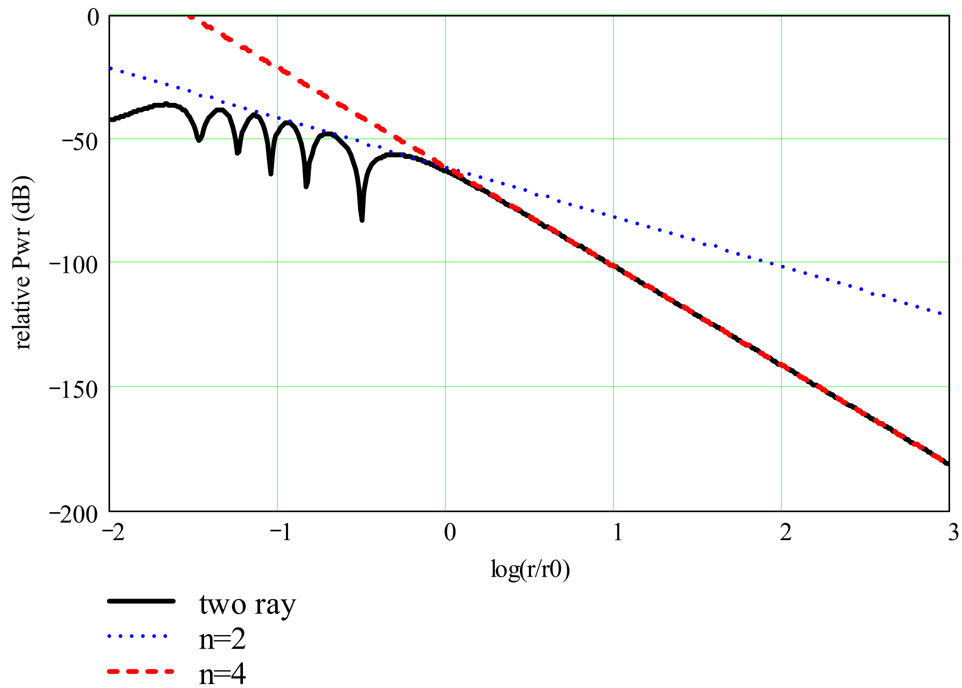

Fig. 3 Ground reflection model (5) (solid curve), envelope of (8) (dotted line), and (9) (dashed line) for h_t=10λ, h_r=3λ. The breakpoint is at 188λ.

5 Experiment

A 915-MHz transmitter was placed 2.5 ft above a sidewalk. Field strength measurements were made at various distances and at a height of 3 ft. The observed data are show in Fig. 4 along with (5) and the free-space model. A reference distance of $r_0=5\,\text{ft}$ was used. The ground-reflection model does capture the general behavior of the data. In Fig. 5 we plot the observed data along with $1/r^n$ models for $n=2,3,4$. This illustrates the breakpoint concept. The theoretical break point of (11) is about 48 feet in this case. Since $\log(48/5)=0.98$ we would expect a breakpoint near 1 on the horizontal axis. This is indeed what we observe. The data are fairly well described by $n=2$ for small *r* and by $n=3$ for large *r*.

6 References

1. Rappaport, T. S., *Wireless Communications: Principles and Practice*, Prentice Hall, 1996, ISBN 0-13-375536-3.
2. Ishimaru, A., *Electromagnetic Wave Propagation, Radiation, and Scattering*, Prentice Hall, 1996, ISBN 0-13-249053-6.

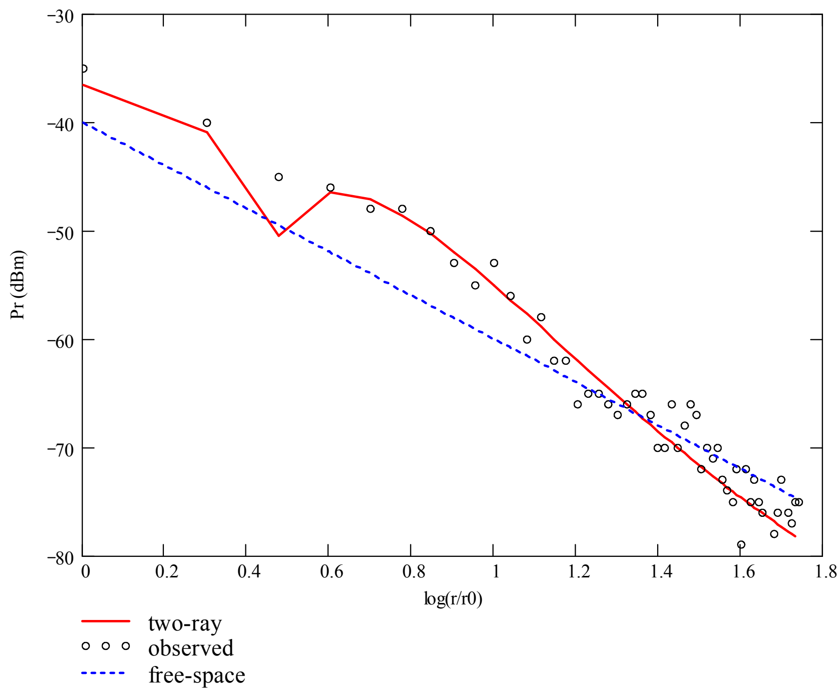

Fig. 4 Ground reflection measurements above a sidewalk at 915MHz. Reference distance r_0 is 5ft. Circles are observed data. Solid curve is (5) with a Γ of –0.6. Dashed curve is free-space model.

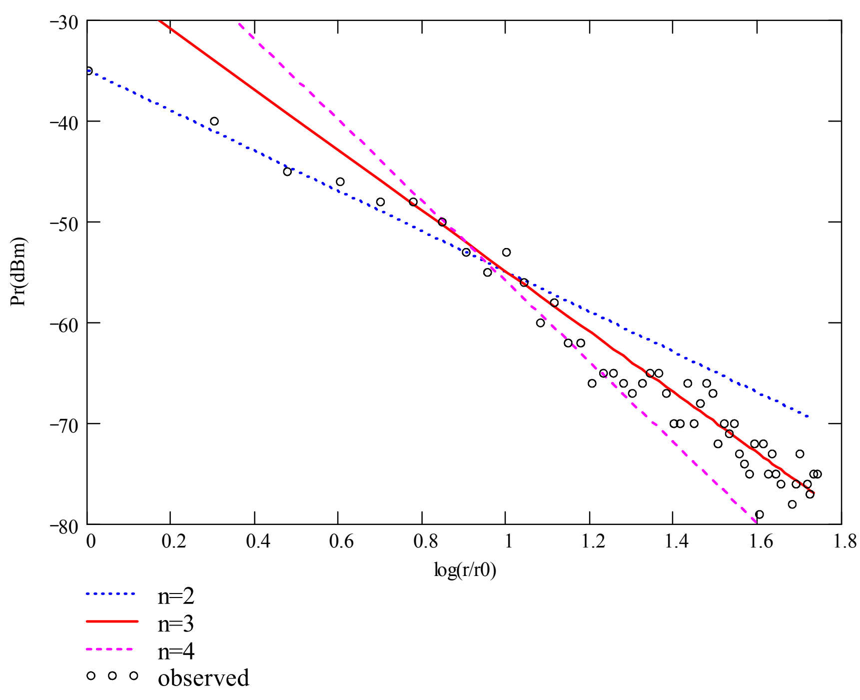

Fig. 5 Data of Fig. 4 compared to various $1/r^n$ models. $n=2$ gives a descent description for small r while $n=3$ gives a descent description of large r.