# Lecture 2

*Radiowave Propagation*

## 1 Introduction

Wireless communication relies on electromagnetic waves to carry information from one point to another. This propagation forms our physical channel. It is essential that we understand it if we are going to be able to understand and design wireless systems. This is a complex subject and will be our focus for several lectures. We will begin by considering the case of free-space propagation. First, however, we need to learn/review the dBm logarithmic power scale, as we will use this extensively.

## 2 Logarithmic Power Scales

Power levels in radio systems can have very large dynamic ranges. For example, a CB ("citizen's band") radio might transmit 4W and receive as little as 10 fW ( $10^{-14}$ W ). That's a dynamic range of more than 14 orders of magnitude. Instead of having to use factors like $10^{14}$, it is usually more convenient to work on a logarithmic scale. Also, most of the systems we will consider can be treated as a cascade of blocks in which each block adds a gain factor. Since logarithms turn factors into sums, working on a log scale often simplifies analysis – you add instead of multiply.

A dimensionless number A can be represented on a decibel scale as

$$\begin{aligned} A_{\mathrm{dB}} &= 10 \log A \\ A &= 10^{A_{\mathrm{dB}}/10} \end{aligned} \tag{1}$$

If $P$ is a power level then it has dimensions. Therefore, we cannot directly convert it to a dB scale. But we can get a dimensionless number by taking the ratio of $P$ to some reference power. The most commonly used reference in RF telecom is one milliwatt, and we write dBm, meaning dB relative to a milliwatt. The relation is

$$\begin{aligned} P_{\mathrm{dBm}} &= 10 \log P_{\mathrm{mW}} \\ P_{\mathrm{mW}} &= 10^{P_{\mathrm{dBm}}/10} \end{aligned} \tag{2}$$

where $P_{\mathrm{mW}}$ is the power level expressed in milliwatts and $P_{\mathrm{dBm}}$ is the corresponding expression in dBm. Sometimes a reference of 1 W is used in which case we write dBW, meaning dB relative to a watt. Then the relation is

$$\begin{aligned} P_{\mathrm{dBW}} &= 10 \log P_{\mathrm{W}} \\ P_{\mathrm{W}} &= 10^{P_{\mathrm{dBW}}/10} \end{aligned} \tag{3}$$

Where $P_{\mathrm{W}}$ is the power expressed in watts. There are numerous other references that can be used, but we will almost always use the dBm scale.

The majority of radio systems operate with a characteristic impedance of 50 Ω. In this case the relationship between power and peak voltage is

$$P=\frac{V^2}{2\cdot 50} \quad V=10\sqrt{P} \tag{4}$$

Where $P$ is power (in watts) and $V$ is voltage (in volts). The following table lists some power levels in dBm and Watts. It also lists the corresponding peak voltage in a 50 Ω system.

| P(dBm) | P | V |
|---|---|---|
| 40 | 10 W | 31.6 V |
| 30 | 1 W | 10 V |
| 0 | 1 mW | 316 mV |
| -30 | 1 μW | 10 mV |
| -60 | 1 nW | 316 μV |
| -90 | 1 pW | 10 μV |
| -110 | 10 fW | 1 μV |
| -120 | 1 fW | 316 nV |

You should develop the ability to "think in dB" without a calculator. Here are some important numbers: 10dB is a factor of 10, 3dB is (very nearly) a factor of 2, and 1dB is (nearly) a factor of 5/4. Likewise: −10dB is a factor of 1/10, −3dB is (very nearly) a factor of 1/2, and −1dB is (nearly) a factor of 4/5.

*Example* 1: To figure out a power level of –47dBm in watts we can write $-47\,\text{dBm}=-50\,\text{dBm}+3\,\text{dB}$ so we have $(10^{-5}\,\text{mW})\cdot 2=20\,\text{nW}$ .

A system has a gain of 15dB. Since 15=10+3+3−1 this gives a factor of $10\cdot 2\cdot 2\cdot\frac{4}{5}=32$ .

It is very important to keep in mind that you cannot add dBm values. For example it is *not* correct to write $10\,\text{dBm}+3\,\text{dBm}=13\,\text{dBm}$ . This implies that you have 10 dBm of power, add 3 dBm of power and end up with 13dBm of power. In fact 10 dBm is 10mW while 3 dBm is 2mW. The sum is 12mW which corresponds to 10.8 dBm. On the other hand $10\,\text{dBm}+3\,\text{dB}=13\,\text{dBm}$ is a correct expression. This means that we have a power level of 10 dBm which is then increased by 3 dB to get 13 dBm of power. On a linear scale we would write $10\,\text{mW}\cdot 2=20\,\text{mW}$ and 20 mW is 13 dBm. In other words, the 10 dBm is put through a system with a power gain of 3 dB (factor of 2) resulting in 13 dBm output power.

# 3 Free-Space Propagation

Suppose we have a radio that transmits power $P_T$ watts *isotropically*, that is, it generates a spherical wave that sends the same power in all directions. At a distance *r* the power $P_T$ will have spread over a sphere of surface area $4\pi r^2$ , so the intensity is $P_T/(4\pi r^2)$ watts per square meter. In practice systems don't transmit isotropically but instead use antennas with *directivity* that increase the power in certain directions and decrease it in others. The *antenna gain*, $G_T$ , is the factor by which this process increases the intensity in some desired direction relative the isotropic case. The intensity in that direction is then $P_T G_T/(4\pi r^2)$ . Sometimes the factor

$P_T G_T$ is referred to as EIRP (Effective Isotropic Radiated Power). This is the power a fictitious system would have to radiate isotropically to get the same intensity the real system (with antenna gain) provides.

Suppose we collect some of this power with an aperture of area $A_R$. The received power will then be the intensity times the area or

$$P_R = P_T \frac{G_T A_R}{4\pi r^2} \tag{5}$$

This is one form of the *Friis equation*. Antenna theory tells us that area and gain are related by $A = G\lambda^2/4\pi$, with λ the wavelength, so we can write

$$P_R = P_T \frac{G_T G_R \lambda^2}{(4\pi r)^2} \tag{6}$$

where $G_R$ is the gain of the receiving antenna. (Note that $\lambda(\text{m}) = 300/f(\text{MHz})$.) This is the form we will make the most use of as antennas are usually characterized by gain instead of area. Expressing the powers in milliwatts and taking $10\log()$ of both sides we get

$$P_{R,dBm} = P_{T,dBm} + G_{T,dB} + G_{R,dB} - 20\log\frac{4\pi r}{\lambda} \tag{7}$$

(Sometimes antenna gain is written as dBi, meaning dB relative to isotropic.) Say we have a received power of $P_0$ at some reference distance $r_0$. We might obtain $P_0$ from (7) as

$$P_0 = P_{T,dBm} + G_{T,dB} + G_{R,dB} - 20\log\frac{4\pi r_0}{\lambda} \tag{8}$$

or we could measure it. In either case, for any other distance we can then write

$$P_{R,dBm} = P_0 - 20\log\frac{r}{r_0} \tag{9}$$

**Exercise** 1: Derive (9) from (7) and (8). *Hint*: in (7) write $r = r_0(r/r_0)$.

Equation (9) is convenient in that we don't have to explicitly deal with the antenna gains and wavelength terms. And, if we treat $\log(r/r_0)$ as the independent variable, then this is a simple slope-intercept linear relationship. Since $\log(10r/r_0) = \log 10 + \log(r/r_0) = 1 + \log(r/r_0)$, when $r$ increases by a factor of 10 (a *decade*), $\log(r/r_0)$ increases by 1, and $P_{R,dBm}$ decreases $-20\,\text{dBm}$. We say that expression (9) has a slope of $-20\,\text{dB/decade}$.

*Example* 2: We transmit 4 W of power at a frequency of 27 MHz (e.g., a CB radio). The corresponding wavelength is 11 m. Let the transmitter and receiver antennas both have gains of 2 (fairly typically for a "dipole" antenna). At a distance of 10 km the received power would be $4\text{W}(2)(2)(11\text{m})^2/(4\pi 10000\text{m})^2$ which gives us 123 nW or –39 dBm. For this received power the voltage level would be 3.5 mV. This might sound like a very weak signal, but even an inexpensive CB radio will have a sensitivity of around 1-μV which corresponds to 10 fW or –110 dBm, so it's about a factor ten million stronger than the weakest signal the radio could pick up. (It's the ability of radios to receiver such small signals that enables radio communication over useful distances.)

For other distances we can then write $P_{R,dBm} = -39 - 20\log\frac{r}{10\text{km}}$. Say we want to find the distance at which $P_{R,dBm} = -100\text{dBm}$. The answer is $r = 10\,\text{km}\left(10^{\frac{-100+39}{-20}}\right) = 11{,}200\,\text{km}$. Now, this result is only valid for an ideal system in free space. In practice the presence of terrain has a *very large* effect on propagation. (We will consider the effects of terrain on radio propagation in future lectures.)

*Example* 3: An example of nearly ideal free-space propagation is provided by satellite communication systems. Say a TV satellite in geostationary orbit (36,000 km altitude), transmits 10 W (40 dBm), operates at 3.95 GHz (76 mm wavelength), and has antenna gain of 27 dBi, which is roughly the directivity needed to spread its transmission over North America. If you want a received signal strength of at least –90 dBm on the ground, what size of antenna do you need?

We need to have $-90 = 40 + 27 + G_{R,dB} - 20\log\frac{4\pi(36{,}000\,\text{km})}{76}\,\text{mm}$. Solving for the gain we get $G_R = 38\,\text{dB}$. On a linear scale this is $G_R = 6310$, so $G\lambda^2/4\pi = 2.9\,\text{m}^2$ is the required aperture area. This gives us a circular dish of diameter 1.9 m.

*Example* 4: The Voyager spacecraft has a 3.7 m-diameter antenna, a 23-W transmitter (44 dBm), and operates at a frequency of 8.4 GHz (36-mm wavelength). The NASA Deep Space Network receiving antennas have diameters of 34 m. The corresponding antenna gains are $G_T = 50\text{dB}$ and $G_R = 70\text{dB}$. We thus have $P_{r,dBm} = 44 + 50 + 70 - 20\log(4\pi r/36\,\text{mm})$. In the year 2000, Voyager's distance from Earth was about 12 billion km. Plugging in this distance we find $P_R = -148\text{dBm}$ or about a millionth of a trillionth of a watt. To get some idea of how small this power level is, consider that a AA battery can hold about 1W-hr of energy. If you drained a AA battery continuously at –148dBm of power it would last several thousands of times the age of the universe before going dead. Yet the sensitivity of this system is such that reliable communication is possible at this signal level.

*Example* 5: Consider an even more extreme example. The Arecibo radio/radar telescope in Puerto Rico can transmit 1 MW (90 dBm) at 2380 MHz (12.6-cm wavelength) and has an antenna gain of about 77 dB. For a 1-Hz bandwidth signal it has a receiving sensitivity of around –180 dBm. Assume that this system transmits a signal to an identical system. Solving

$$-180 = 90 + 77 + 77 - 20\log(4\pi r/12.6\text{cm})$$

we find that $r$ is about 1700 light-years. It is numbers such as these that encourage intelligent people to spend time looking for signs of extra terrestrial intelligence in the radio spectrum.

# 4 References

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2. Ramo, S., J. R. Whinnery, T. V. Duzer, Fields and Waves in Communication Electronics, Wiley, 1994, ISBN 0-471-58551-3.