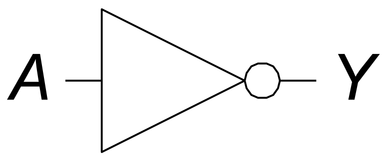
Logic Gates

- **Perform logic functions:**
  - inversion (NOT), AND, OR, NAND, NOR, etc.
- **Single-input:**
  - NOT gate, buffer
- **Two-input:**
  - AND, OR, XOR, NAND, NOR, XNOR
- **Multiple-input**

Single-Input Logic Gates

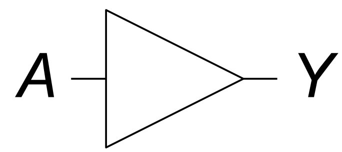
NOT



$$Y = \overline{A}$$

A	Y
0	
1	

BUF

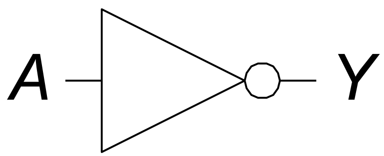


$$Y = A$$

A	Y
0	
1	

Single-Input Logic Gates

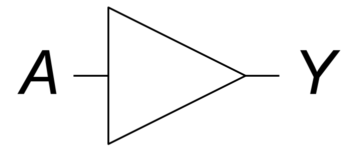
NOT



$$Y = \overline{A}$$

A	Y
0	1
1	0

BUF



$$Y = A$$

A	Y
0	0
1	1

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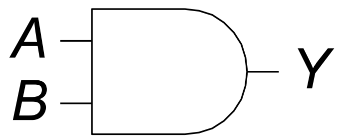
Two-Input Logic Gates

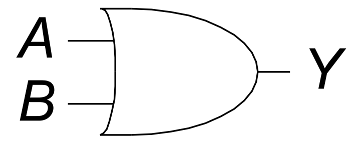
AND



$Y = AB$

A	B	Y
0	0	
0	1	
1	0	
1	1	

OR

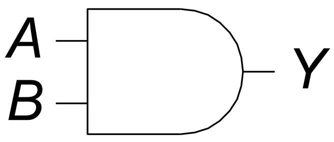


$Y = A + B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

Two-Input Logic Gates

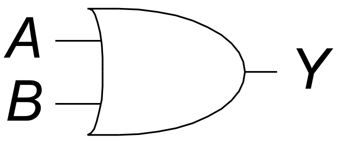
AND



$$Y = AB$$

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

OR



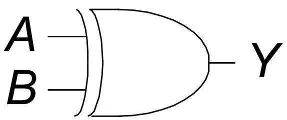
$$Y = A + B$$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	1

More Two-Input Logic Gates

FROM ZERO TO ONE

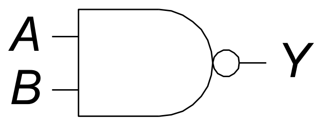
XOR



$Y = A \oplus B$

A	B	Y
0	0	
0	1	
1	0	
1	1	

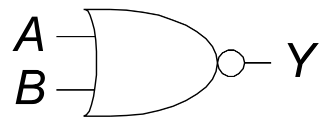
NAND



$Y = \overline{AB}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

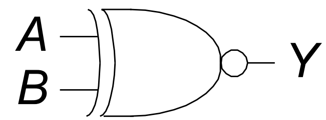
NOR



$Y = \overline{A + B}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

XNOR

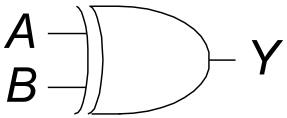


$Y = \overline{A \oplus B}$

A	B	Y
0	0	
0	1	
1	0	
1	1	

More Two-Input Logic Gates

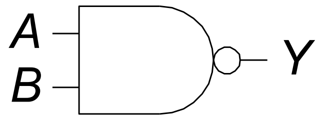
XOR



$Y = A \oplus B$

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

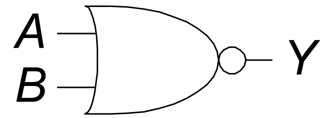
NAND



$Y = \overline{AB}$

A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

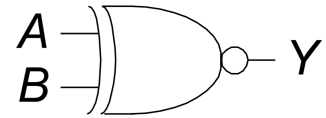
NOR



$Y = \overline{A + B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

XNOR



$Y = \overline{A \oplus B}$

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

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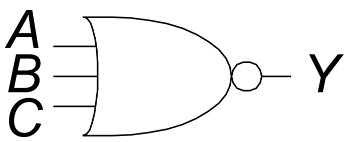
Multiple-Input Logic Gates

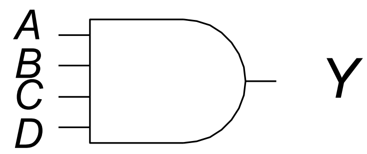
NOR3



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

AND4

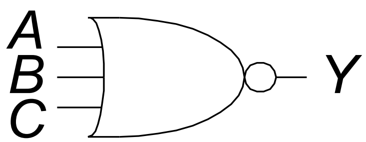


$Y = ABCD$

A	B	C	Y
0	0	0	
0	0	1	
0	1	0	
0	1	1	
1	0	0	
1	0	1	
1	1	0	
1	1	1	

Multiple-Input Logic Gates

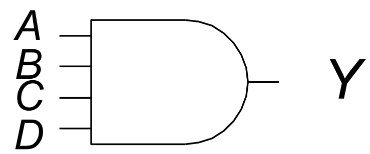
NOR3



$Y = \overline{A+B+C}$

A	B	C	Y
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

AND4



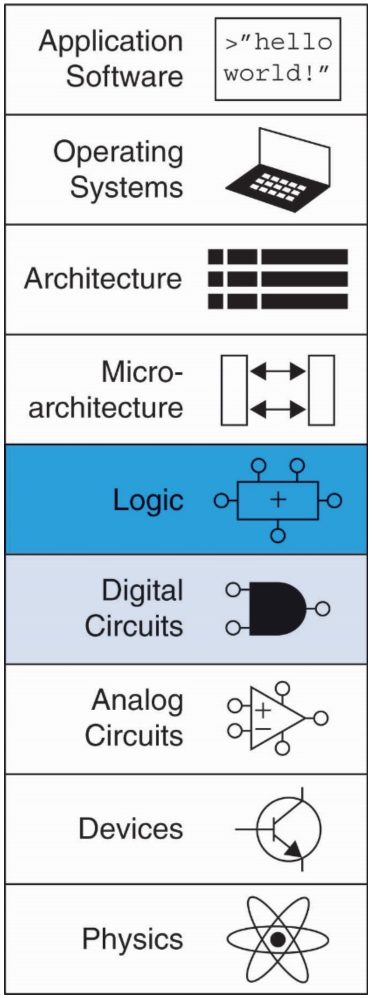
$Y = ABCD$

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

- Multi-input XOR: Odd parity

Chapter 2 :: Topics

- **Introduction**
- **Boolean Equations**
- **Boolean Algebra**
- **From Logic to Gates**
- **Multilevel Combinational Logic**
- **X's and Z's, Oh My**
- **Karnaugh Maps**
- **Combinational Building Blocks**
- **Timing**



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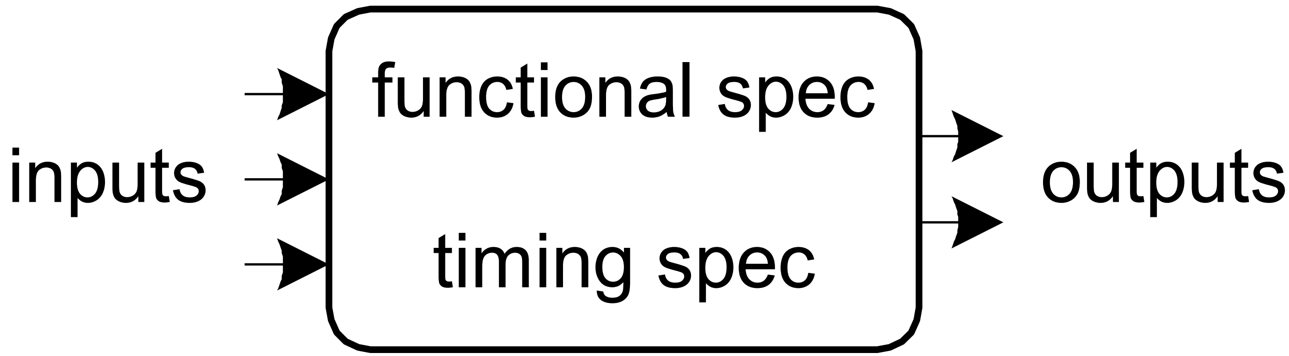
Introduction

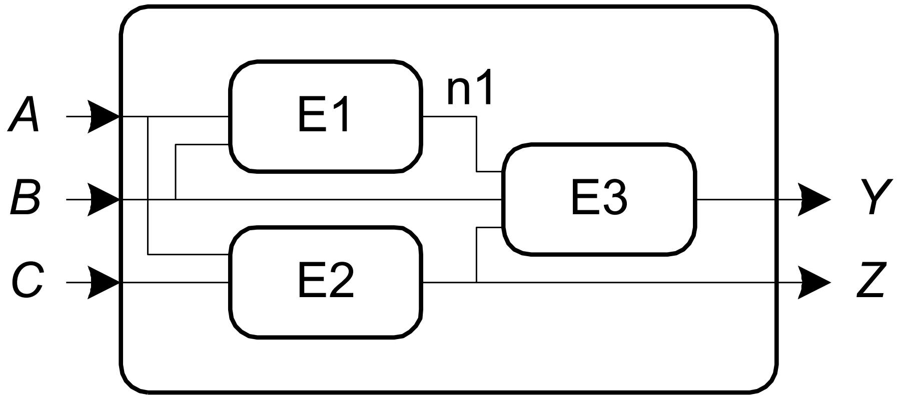
A logic circuit is composed of:

- Inputs
- Outputs
- Functional specification
- Timing specification



Circuits

- Nodes
  - Inputs: *A*, *B*, *C*
  - Outputs: *Y*, *Z*
  - Internal: n1
- Circuit elements
  - E1, E2, E3
  - Each a circuit



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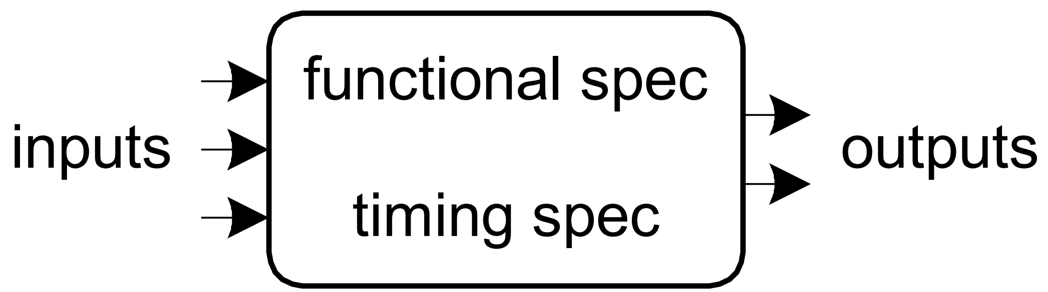
Types of Logic Circuits

- **Combinational Logic**
  - Memoryless
  - Outputs determined by current values of inputs
- **Sequential Logic**
  - Has memory
  - Outputs determined by previous and current values of inputs



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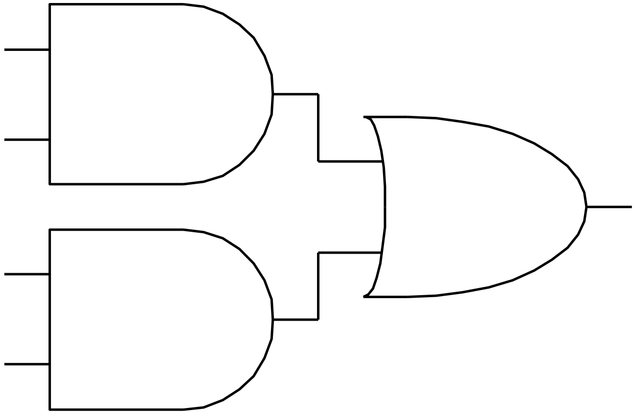
Rules of Combinational Composition

- Every element is combinational
- Every node is either an input or connects to *exactly one* output
- The circuit contains no cyclic paths
- **Example:**



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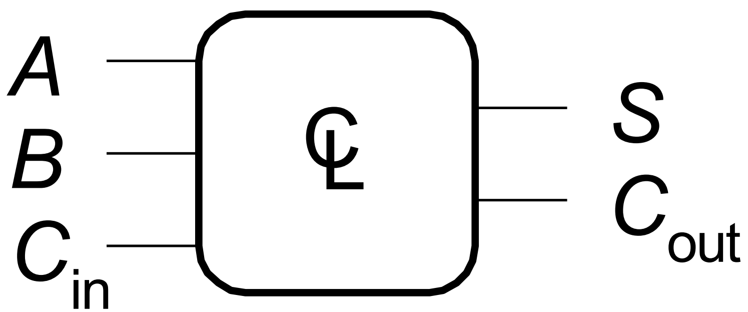
Boolean Equations

- Functional specification of outputs in terms of inputs
- **Example:** $S = F(A, B, C_{in})$

  $C_{out} = F(A, B, C_{in})$



$$S = A \oplus B \oplus C_{in}$$
$$C_{out} = AB + AC_{in} + BC_{in}$$

Some Definitions

- Complement: variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$

- Literal: variable or its complement

  $A, \bar{A}, B, \bar{B}, C, \bar{C}$

- Implicant: product of literals

  $AB\bar{C}, \bar{A}C, BC$

- Minterm: product that includes all input variables

  $AB\bar{C}, A\bar{B}\bar{C}, ABC$

- Maxterm: sum that includes all input variables

  $(A+\bar{B}+C), (\bar{A}+B+\bar{C}), (\bar{A}+\bar{B}+C)$

Sum-of-Products (SOP) Form

- All equations can be written in SOP form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	B	Y	minterm	minterm name
0	0	0	$\overline{A}\ \overline{B}$	m_0
0	1	1	$\overline{A}\ B$	m_1
1	0	0	$A\ \overline{B}$	m_2
1	1	1	$A\ B$	m_3

$$Y = \mathbf{F}(A, B) =$$

FROM ZERO TO ONE

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Sum-of-Products (SOP) Form

- All equations can be written in SOP form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	***B***	***Y***	**minterm**	**minterm name**
0	0	0	$\bar{A}\,\bar{B}$	m_0
0	1	1	$\bar{A}\,B$	m_1
1	0	0	$A\,\bar{B}$	m_2
1	1	1	$A\,B$	m_3

$$Y = F(A, B) =$$

FROM ZERO TO ONE
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Sum-of-Products (SOP) Form

- All equations can be written in SOP form
- Each row has a **minterm**
- A minterm is a product (AND) of literals
- Each minterm is TRUE for that row (and only that row)
- Form function by ORing minterms where the output is TRUE
- Thus, a sum (OR) of products (AND terms)

A	*B*	*Y*	minterm	minterm name
0	0	0	$\overline{A}\,\overline{B}$	m_0
0	1	1	$\overline{A}\,B$	m_1
1	0	0	$A\,\overline{B}$	m_2
1	1	1	$A\,B$	m_3

$$Y = \mathrm{F}(A, B) = \overline{\mathrm{A}}\mathrm{B} + \mathrm{AB} = \Sigma(1, 3)$$

FROM ZERO TO ONE
ELSEVIER

Product-of-Sums (POS) Form

- All Boolean equations can be written in POS form
- Each row has a **maxterm**
- A maxterm is a sum (OR) of literals
- Each maxterm is FALSE for that row (and only that row)
- Form function by ANDing the maxterms for which the output is FALSE
- Thus, a product (AND) of sums (OR terms)

A	B	Y	maxterm	maxterm name
0	0	0	$A + B$	M_0
0	1	1	$A + \overline{B}$	M_1
1	0	0	$\overline{A} + B$	M_2
1	1	1	$\overline{A} + \overline{B}$	M_3

$$Y = \mathbf{F}(A, B) = (A + B)(A + \overline{B}) = \Pi(0, 2)$$

correction: not(A) + B

Boolean Equations Example

- You are going to the cafeteria for lunch
  - You won't eat lunch (E)
  - If it's not open (O) or
  - If they only serve corndogs (C)
- Write a truth table for determining if you will eat lunch (E).

O	*C*	*E*
0	0	
0	1	
1	0	
1	1	

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Boolean Equations Example

- You are going to the cafeteria for lunch
  - You won't eat lunch (E: "will eat")
  - If it's not open (O: "it's open") or
  - If they only serve corndogs (C: "corndogs only")
- Write a truth table for determining if you will eat lunch (E).

O	C	E
0	0	0
0	1	0
1	0	1
1	1	0

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SOP & POS Form

- SOP – sum-of-products

O	*C*	*E*	minterm
0	0		$\overline{O}\ \overline{C}$
0	1		$\overline{O}\ C$
1	0		$O\ \overline{C}$
1	1		$O\ C$

- POS – product-of-sums

O	*C*	*Y*	maxterm
0	0		$O + C$
0	1		$O + \overline{C}$
1	0		$\overline{O} + C$
1	1		$\overline{O} + \overline{C}$

SOP & POS Form

- SOP – sum-of-products

O	C	E	minterm
0	0	0	$\overline{O}\ \overline{C}$
0	1	0	$\overline{O}\ C$
1	0	1	$O\ \overline{C}$
1	1	0	$O\ C$

$$Y = O\overline{C}$$
$$= \Sigma(2)$$

- POS – product-of-sums

O	C	E	maxterm
0	0	0	$O + C$
0	1	0	$O + \overline{C}$
1	0	1	$\overline{O} + C$
1	1	0	$\overline{O} + \overline{C}$

$$Y = (O + C)(O + \overline{C})(\overline{O} + \overline{C})$$
$$= \Pi(0, 1, 3)$$

Boolean Algebra

- Axioms and theorems to **simplify** Boolean equations
- Like regular algebra, but simpler: variables have only two values (1 or 0)
- **Duality** in axioms and theorems:
  - ANDs and ORs, 0's and 1's interchanged

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Boolean Axioms

	Axiom		Dual	Name
A1	$B = 0$ if $B \neq 1$	A1′	$B = 1$ if $B \neq 0$	Binary field
A2	$\overline{0} = 1$	A2′	$\overline{1} = 0$	NOT
A3	$0 \bullet 0 = 0$	A3′	$1 + 1 = 1$	AND/OR
A4	$1 \bullet 1 = 1$	A4′	$0 + 0 = 0$	AND/OR
A5	$0 \bullet 1 = 1 \bullet 0 = 0$	A5′	$1 + 0 = 0 + 1 = 1$	AND/OR

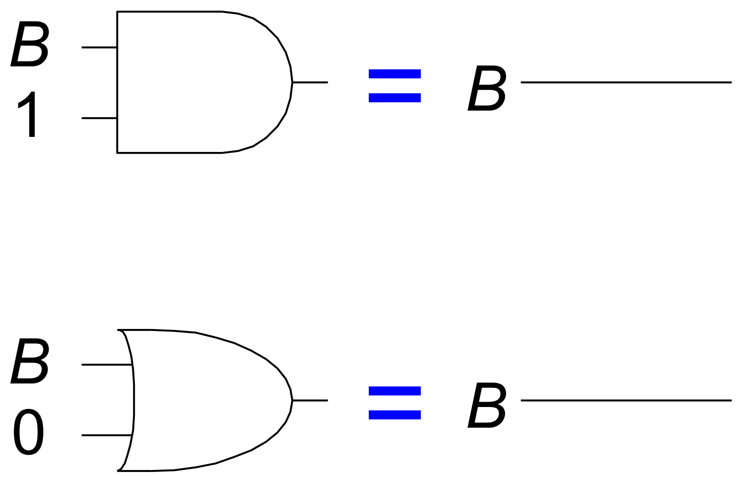
	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\overline{\overline{B}} = B$		Involution
T5	$B \bullet \overline{B} = 0$	T5′	$B + \overline{B} = 1$	Complements

T1: Identity Theorem

- $B \cdot 1 = B$
- $B + 0 = B$

T1: Identity Theorem

- $B \cdot 1 = B$
- $B + 0 = B$

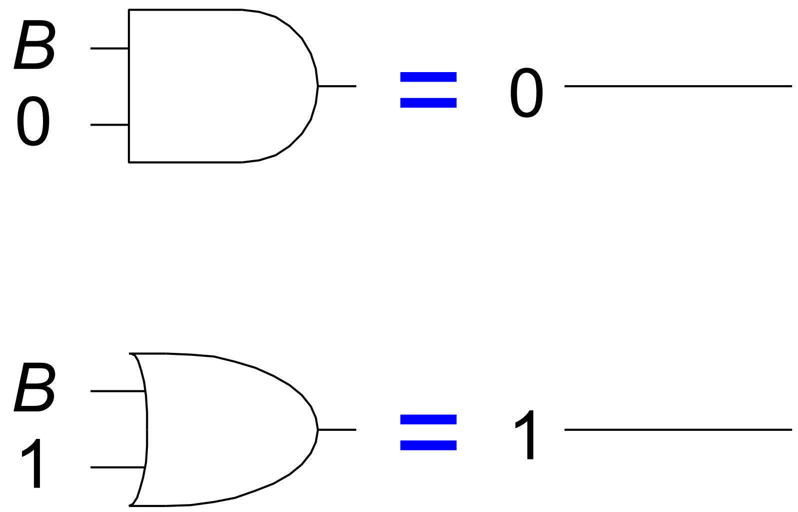


T2: Null Element Theorem

- $B \cdot 0 = 0$
- $B + 1 = 1$

T2: Null Element Theorem

- $B \cdot 0 = 0$
- $B + 1 = 1$



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FROM ZERO TO ONE

T3: Idempotency Theorem

- $B \cdot B = B$
- $B + B = B$

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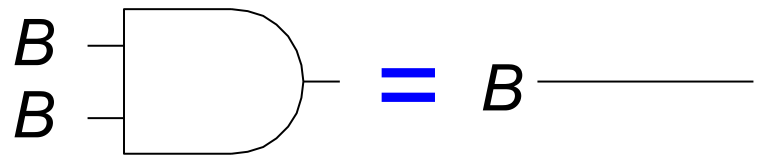
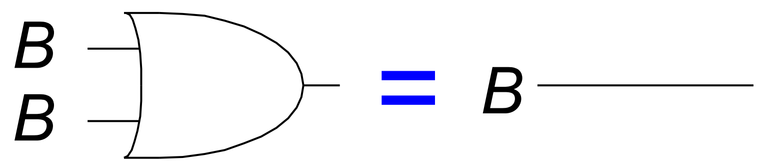
T3: Idempotency Theorem

- $B \bullet B = B$
- $B + B = B$





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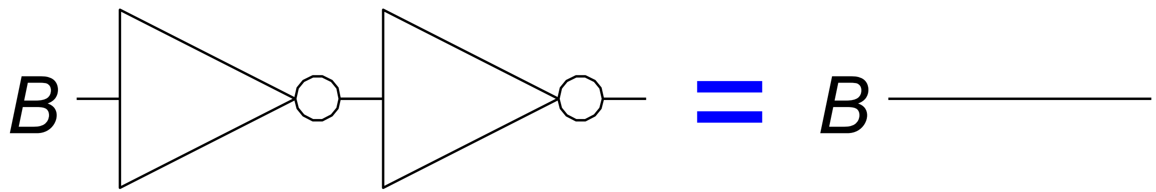
T4: Identity Theorem

- $\overline{\overline{B}} = B$

T4: Identity Theorem

- $\overline{\overline{B}} = B$



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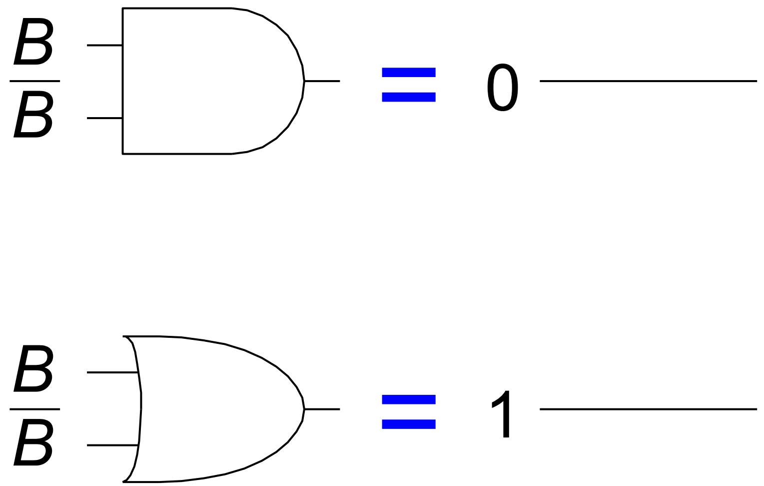
T5: Complement Theorem

- $B \bullet \overline{B} = 0$
- $B + \overline{B} = 1$

T5: Complement Theorem

- $B \bullet \overline{B} = 0$
- $B + \overline{B} = 1$



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Boolean Theorems Summary

	Theorem		Dual	Name
T1	$B \bullet 1 = B$	T1′	$B + 0 = B$	Identity
T2	$B \bullet 0 = 0$	T2′	$B + 1 = 1$	Null Element
T3	$B \bullet B = B$	T3′	$B + B = B$	Idempotency
T4		$\bar{\bar{B}} = B$		Involution
T5	$B \bullet \bar{B} = 0$	T5′	$B + \bar{B} = 1$	Complements

Boolean Theorems of Several Vars

	Theorem		Dual	Name
T6	$B \cdot C = C \cdot B$	T6′	$B + C = C + B$	Commutativity
T7	$(B \cdot C) \cdot D = B \cdot (C \cdot D)$	T7′	$(B + C) + D = B + (C + D)$	Associativity
T8	$(B \cdot C) + B \cdot D = B \cdot (C + D)$	T8′	$(B + C) \cdot (B + D) = B + (C \cdot D)$	Distributivity
T9	$B \cdot (B + C) = B$	T9′	$B + (B \cdot C) = B$	Covering
T10	$(B \cdot C) + (B \cdot \overline{C}) = B$	T10′	$(B + C) \cdot (B + \overline{C}) = B$	Combining
T11	$(B \cdot C) + (\overline{B} \cdot D) + (C \cdot D) = B \cdot C + \overline{B} \cdot D$	T11′	$(B + C) \cdot (\overline{B} + D) \cdot (C + D) = (B + C) \cdot (\overline{B} + D)$	Consensus
T12	$\overline{B_0 \cdot B_1 \cdot B_2 \ldots} = (\overline{B_0} + \overline{B_1} + \overline{B_2} \ldots)$	T12′	$\overline{B_0 + B_1 + B_2 \ldots} = (\overline{B_0} \cdot \overline{B_1} \cdot \overline{B_2})$	De Morgan's Theorem

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Simplifying Boolean Equations

Example 1:

- $Y = AB + \overline{A}B$

Simplifying Boolean Equations

Example 1:

- $Y = AB + \overline{A}B$

$= B(A + \overline{A})$ T8

$= B(1)$ T5'

$= B$ T1

Simplifying Boolean Equations

Example 2:

- $Y = A(AB + ABC)$

Simplifying Boolean Equations

Example 2:

- $Y = A(AB + ABC)$

$= A(AB(1 + C))$	T8
$= A(AB(1))$	T2′
$= A(AB)$	T1
$= (AA)B$	T7
$= AB$	T3

FROM ZERO TO ONE
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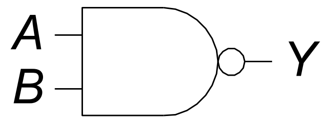
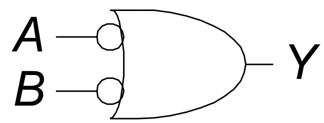
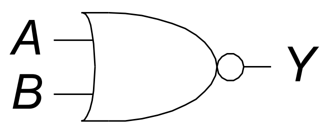
DeMorgan's Theorem

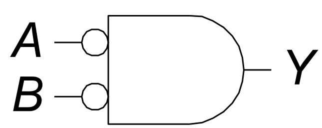
- $Y = \overline{AB} = \overline{A} + \overline{B}$





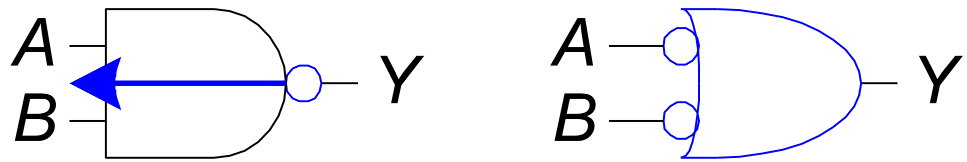
- $Y = \overline{A + B} = \overline{A} \bullet \overline{B}$





Bubble Pushing

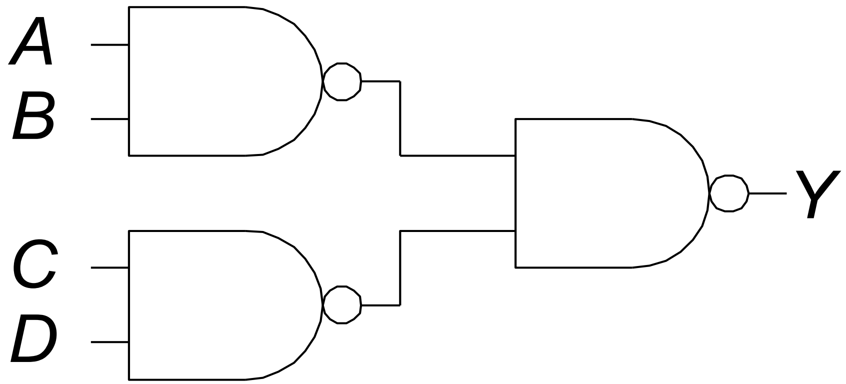
- **Backward:**
  - Body changes
  - Adds bubbles to inputs



- **Forward:**
  - Body changes
  - Adds bubble to output

Bubble Pushing

- What is the Boolean expression for this circuit?



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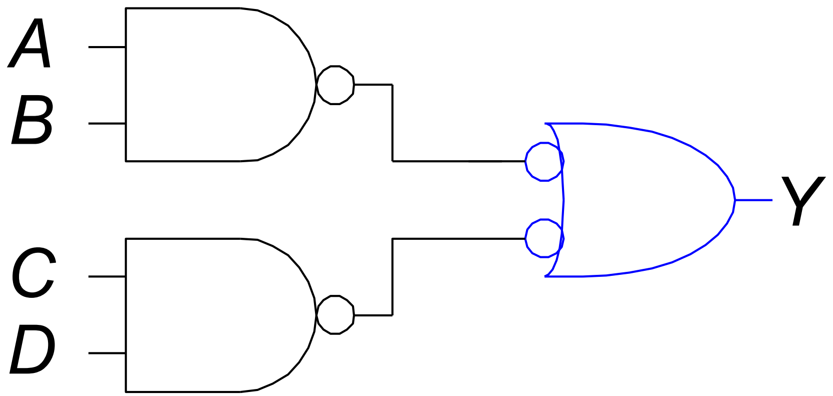
Bubble Pushing

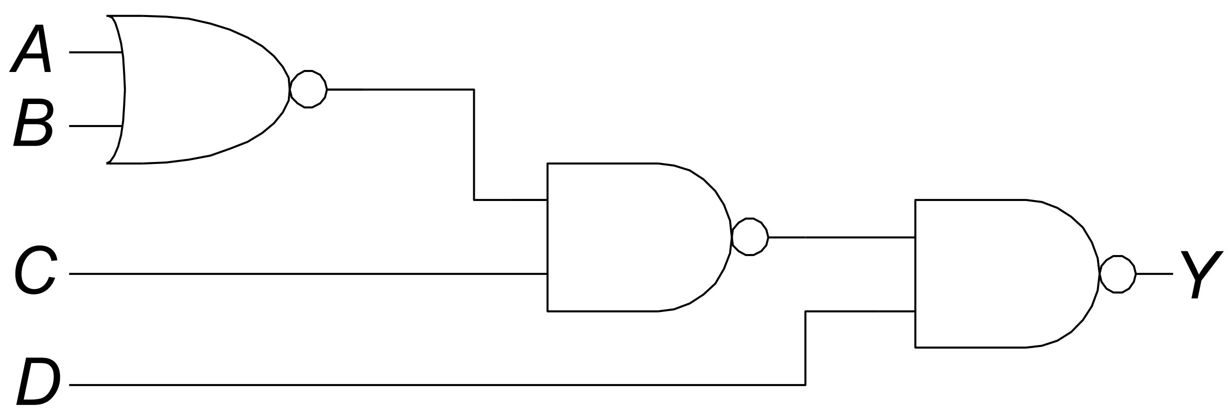
- What is the Boolean expression for this circuit?



$$Y = AB + CD$$

Bubble Pushing Rules

- Begin at output, then work toward inputs
- Push bubbles on final output back
- Draw gates in a form so bubbles cancel



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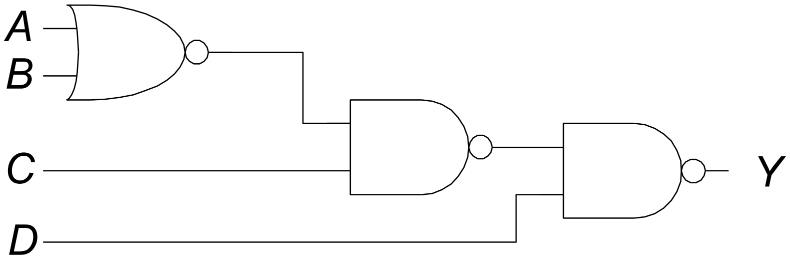
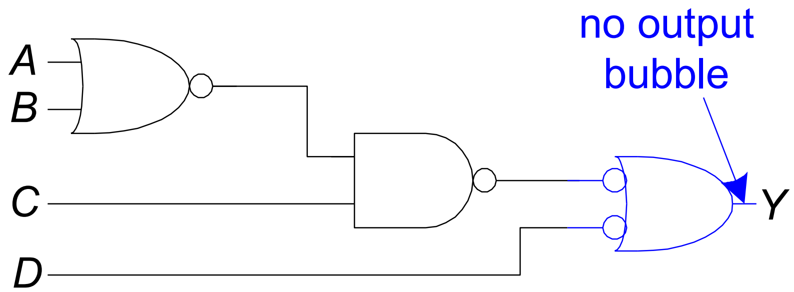
Bubble Pushing Example



Bubble Pushing Example



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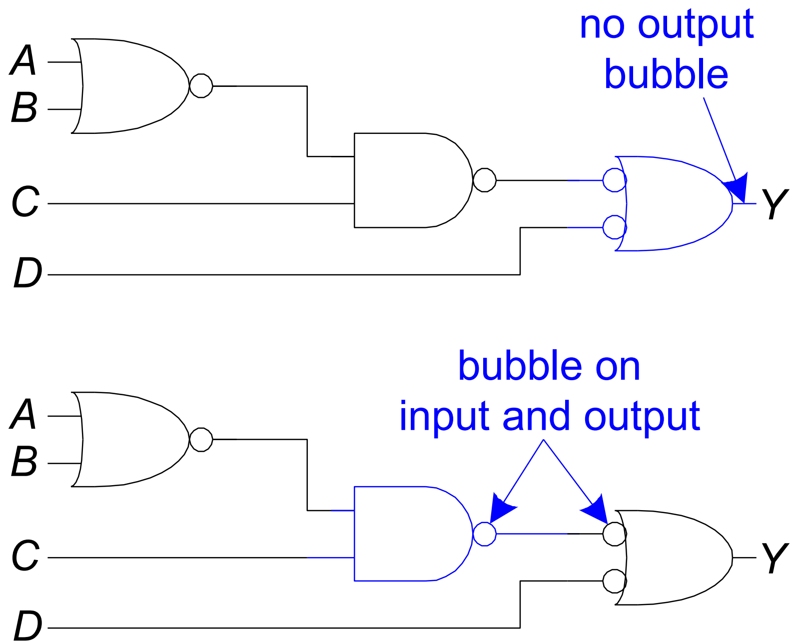
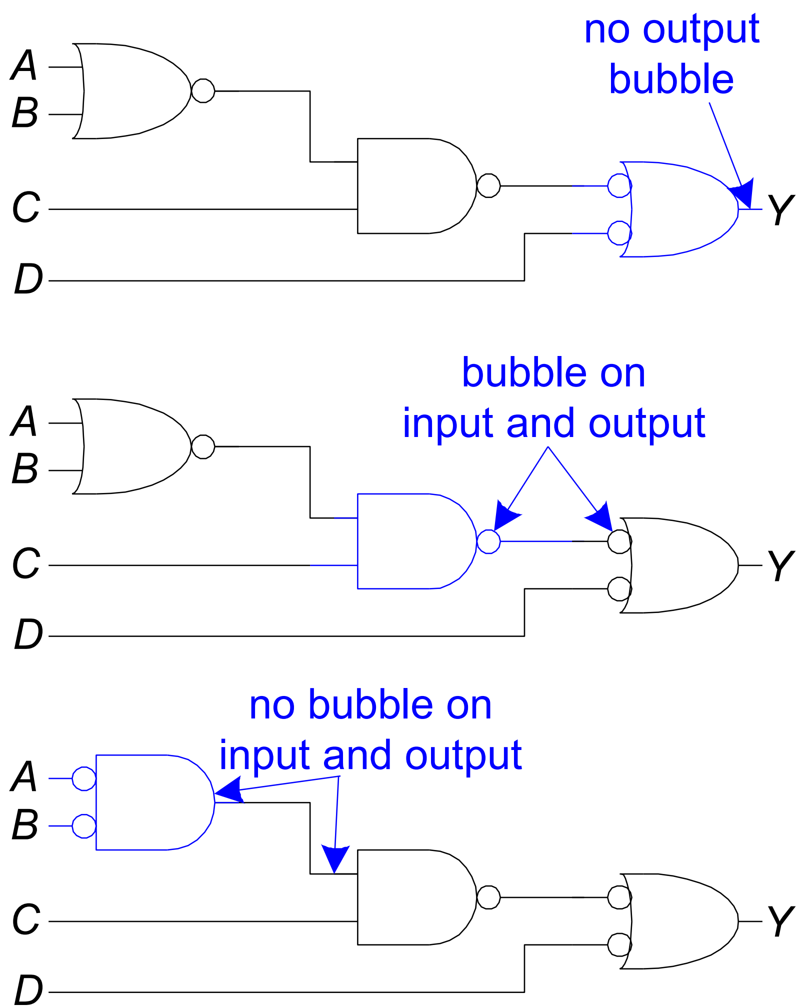
Bubble Pushing Example



FROM ZERO TO ONE

Bubble Pushing Example



$Y = \overline{A}\,\overline{B}C + \overline{D}$

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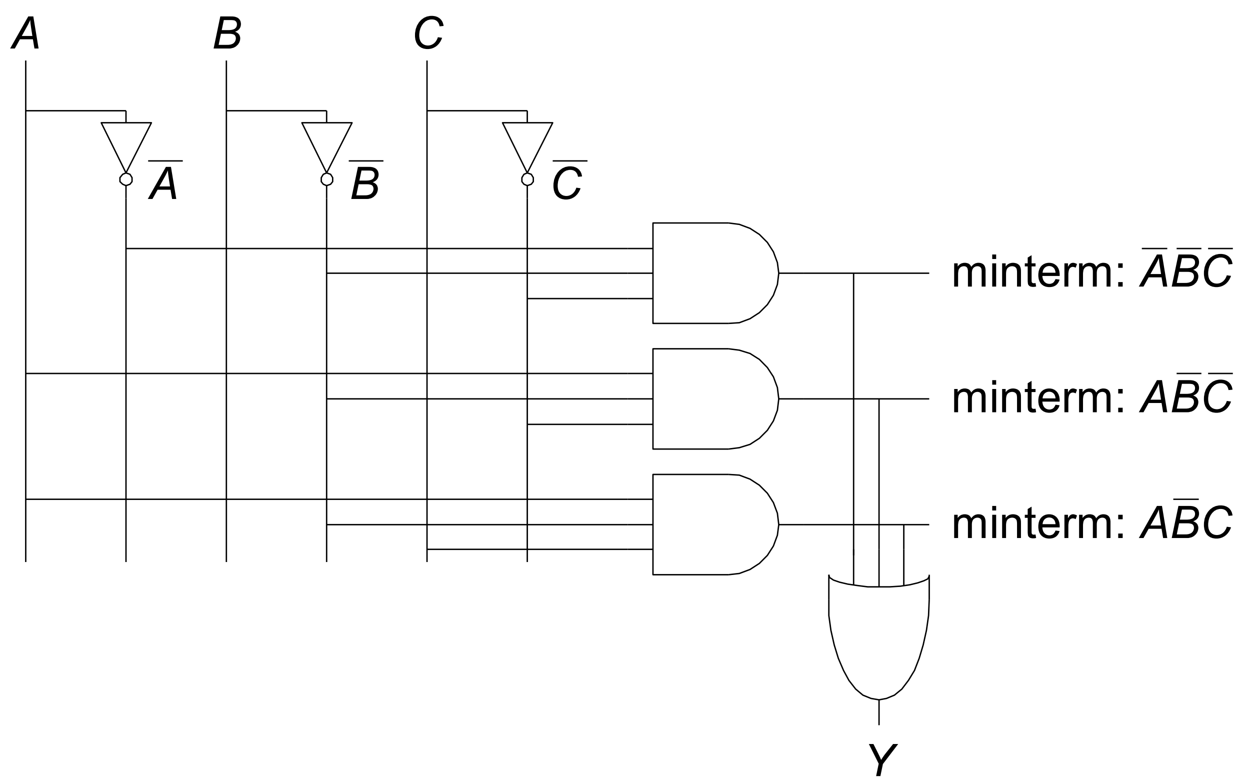
FROM ZERO TO ONE

From Logic to Gates

- Two-level logic: ANDs followed by ORs
- Example: $Y = \bar{A}\bar{B}\bar{C} + A\bar{B}\bar{C} + A\bar{B}C$



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Circuit Schematics Rules

- Inputs on the left (or top)
- Outputs on right (or bottom)
- Gates flow from left to right
- Straight wires are best

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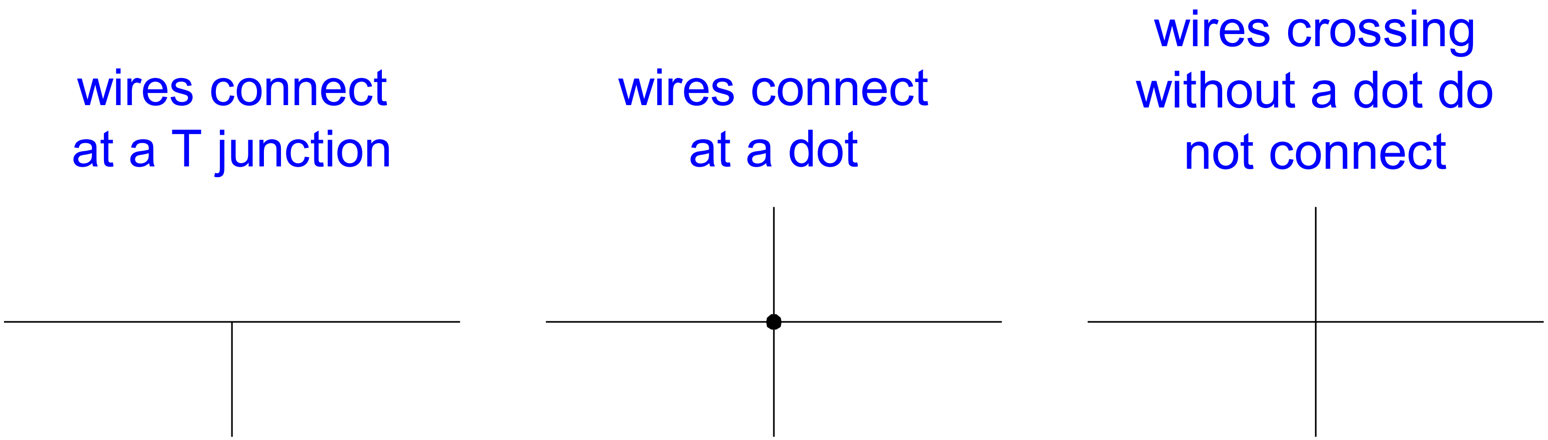
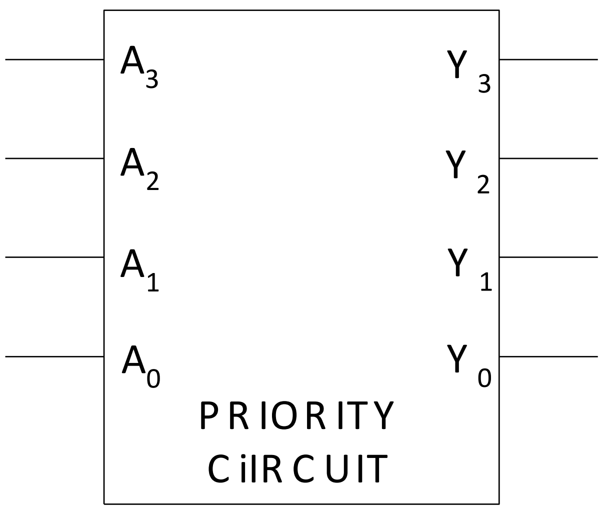
Circuit Schematic Rules (cont.)

- Wires always connect at a T junction
- A dot where wires cross indicates a connection between the wires
- Wires crossing *without* a dot make no connection



Multiple-Output Circuits

- **Example: Priority Circuit**
  Output asserted corresponding to most significant TRUE input

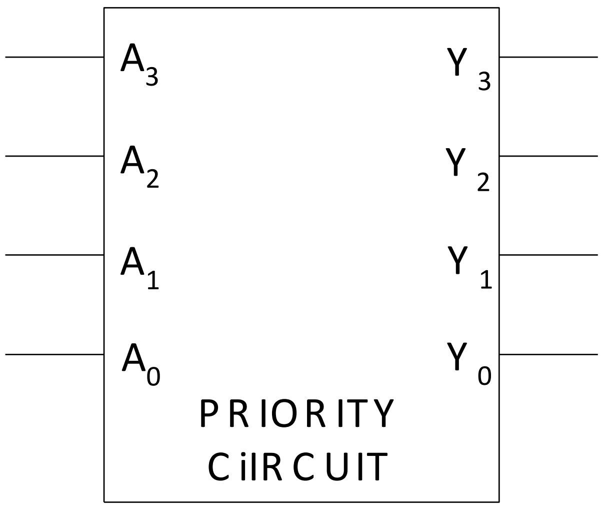


A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0				
0	0	0	1				
0	0	1	0				
0	0	1	1				
0	1	0	0				
0	1	0	1				
0	1	1	0				
0	1	1	1				
1	0	0	0				
1	0	0	1				
1	0	1	0				
1	0	1	1				
1	1	0	0				
1	1	0	1				
1	1	1	0				
1	1	1	1				

Multiple-Output Circuits

- **Example: Priority Circuit**

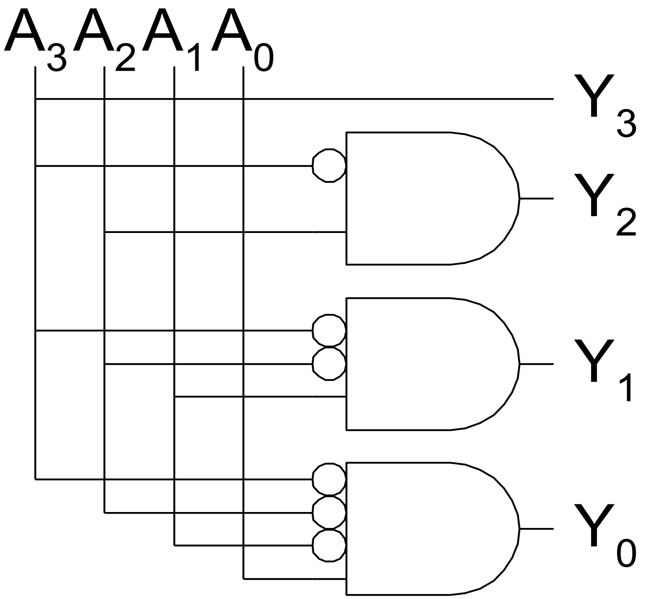
  Output asserted corresponding to most significant TRUE input



A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

Priority Circuit Hardware

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0



FROM ZERO TO ONE

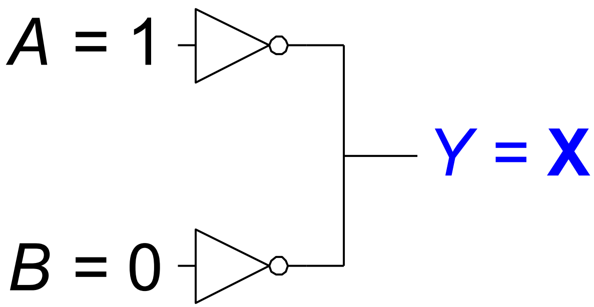
Don't Cares

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	0	0	0	1	0
0	0	1	1	0	0	1	0
0	1	0	0	0	1	0	0
0	1	0	1	0	1	0	0
0	1	1	0	0	1	0	0
0	1	1	1	0	1	0	0
1	0	0	0	1	0	0	0
1	0	0	1	1	0	0	0
1	0	1	0	1	0	0	0
1	0	1	1	1	0	0	0
1	1	0	0	1	0	0	0
1	1	0	1	1	0	0	0
1	1	1	0	1	0	0	0
1	1	1	1	1	0	0	0

A_3	A_2	A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	1
0	0	1	X	0	0	1	0
0	1	X	X	0	1	0	0
1	X	X	X	1	0	0	0

Contention: X

- Contention: circuit tries to drive output to 1 **and** 0
  - Actual value somewhere in between
  - Could be 0, 1, or in forbidden zone
  - Might change with voltage, temperature, time, noise
  - Often causes excessive power dissipation



- **Warnings:**
  - Contention usually indicates a **bug**.
  - **X is used for "don't care" and contention** - look at the context to tell them apart

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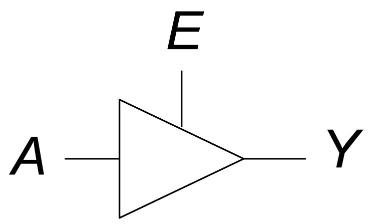
Floating: Z

- Floating, high impedance, open, high Z
- Floating output might be 0, 1, or somewhere in between
  - A voltmeter won't indicate whether a node is floating

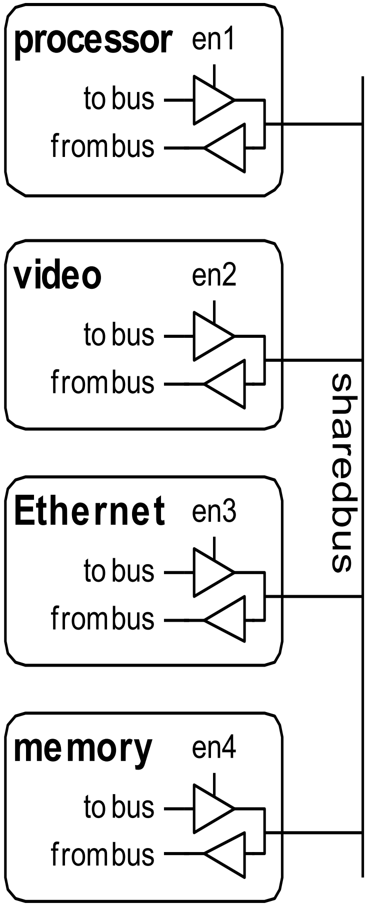
Tristate Buffer



E	A	Y
0	0	Z
0	1	Z
1	0	0
1	1	1

Tristate Busses

- Floating nodes are used in tristate busses
  - Many different drivers
  - Exactly one is active at once



Karnaugh Maps (K-Maps)

- Boolean expressions can be minimized by combining terms
- K-maps minimize equations graphically
- $PA + P\overline{A} = P$

A	B	C	Y
0	0	0	**1**
0	0	1	**1**
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y

C \ AB	00	01	11	10
0	**1**	0	0	0
1	**1**	0	0	0

Y

C \ AB	00	01	11	10
0	**$\overline{A}\overline{B}\overline{C}$**	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\overline{C}$
1	**$\overline{A}\overline{B}C$**	$\overline{A}BC$	ABC	$A\overline{B}C$

K-Map

FROM ZERO TO ONE

- Circle 1's in adjacent squares
- In Boolean expression, include only literals whose true and complement form are ***not*** in the circle

A	B	C	Y
0	0	0	1
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

Y

C \ AB	00	01	11	10
0	1	0	0	0
1	1	0	0	0

$$Y = \overline{A}\,\overline{B}$$

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3-Input K-Map

Y (C \ AB)	00	01	11	10
0	$\bar{A}\bar{B}\bar{C}$	$\bar{A}B\bar{C}$	$AB\bar{C}$	$A\bar{B}\bar{C}$
1	$\bar{A}\bar{B}C$	$\bar{A}BC$	ABC	$A\bar{B}C$

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	**1**
0	1	1	**1**
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	**1**

K-Map

Y (C \ AB)	00	01	11	10
0				
1				

3-Input K-Map

FROM ZERO TO ONE

Y: C \ AB	00	01	11	10
0	$\overline{A}\,\overline{B}\,\overline{C}$	$\overline{A}B\overline{C}$	$AB\overline{C}$	$A\overline{B}\,\overline{C}$
1	$\overline{A}\,\overline{B}C$	$\overline{A}BC$	ABC	$A\overline{B}C$

Truth Table

A	B	C	Y
0	0	0	0
0	0	1	0
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

K-Map

Y: C \ AB	00	01	11	10
0	0	1	~~1~~ 0	0
1	0	1	~~0~~ 1	0

$$Y = \overline{A}B + B\overline{C}$$

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K-Map Definitions

- **Complement:** variable with a bar over it

  $\bar{A}, \bar{B}, \bar{C}$

- **Literal:** variable or its complement

  $\bar{A}, A, \bar{B}, B, C, \bar{C}$

- **Implicant:** product of literals

  $A\bar{B}C, \bar{A}C, BC$

- **Prime implicant:** implicant corresponding to the largest circle in a K-map

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K-Map Rules

- Every 1 must be circled at least once
- Each circle must span a power of 2 (i.e. 1, 2, 4) squares in each direction
- Each circle must be as large as possible
- A circle may wrap around the edges
- A “don't care” (X) is circled only if it helps minimize the equation

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4-Input K-Map

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

Y

CD \ *AB*	00	01	11	10
00				
01				
11				
10				

4-Input K-Map

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0

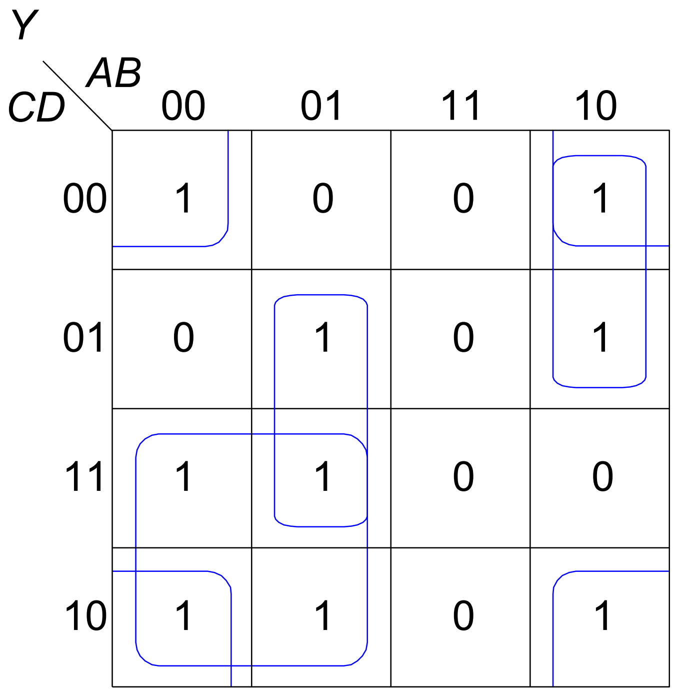
Y

CD \ *AB*	00	01	11	10
00	1	0	0	1
01	0	1	0	1
11	1	1	0	0
10	1	1	0	1

FROM ZERO TO ONE

4-Input K-Map

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	1
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	1
1	0	1	1	0
1	1	0	0	0
1	1	0	1	0
1	1	1	0	0
1	1	1	1	0



$$Y = \overline{A}C + \overline{A}BD + A\overline{B}\overline{C} + \overline{B}\overline{D}$$

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K-Maps with Don't Cares

A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ *AB*	00	01	11	10
00				
01				
11				
10				

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K-Maps with Don’t Cares

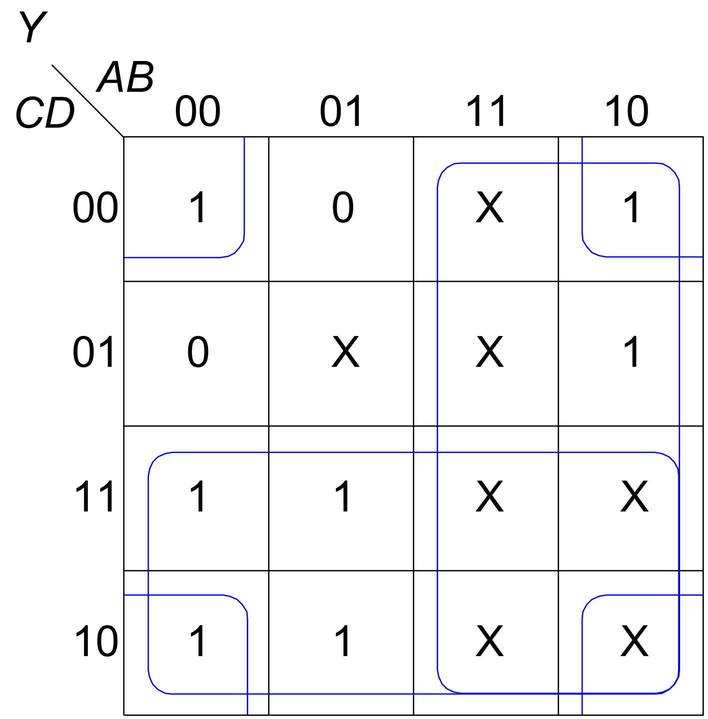
A	*B*	*C*	*D*	*Y*
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X

Y

CD \ *AB*	00	01	11	10
00	1	0	X	1
01	0	X	X	1
11	1	1	X	X
10	1	1	X	X

K-Maps with Don't Cares

A	B	C	D	Y
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	1
0	1	0	0	0
0	1	0	1	X
0	1	1	0	1
0	1	1	1	1
1	0	0	0	1
1	0	0	1	1
1	0	1	0	X
1	0	1	1	X
1	1	0	0	X
1	1	0	1	X
1	1	1	0	X
1	1	1	1	X



$Y = A + \overline{B}\overline{D} + C$

FROM ZERO TO ONE

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Combinational Building Blocks

- Multiplexers
- Decoders

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FROM ZERO TO ONE
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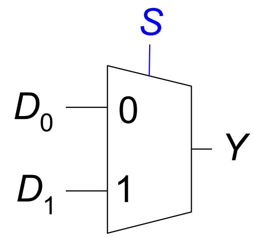
Multiplexer (Mux)

- Selects between one of *N* inputs to connect to output
- $\log_2 N$-bit select input – control input
- **Example:**

2:1 Mux

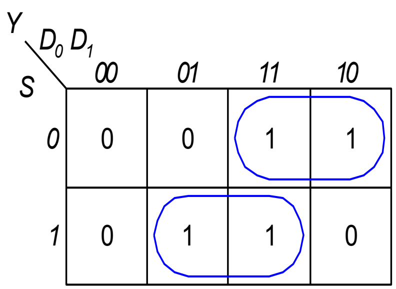


S	D_1	D_0	Y
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	1
1	0	0	0
1	0	1	0
1	1	0	1
1	1	1	1

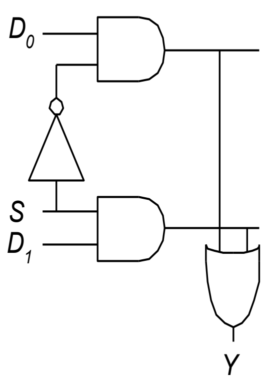
S	Y
0	D_0
1	D_1

Multiplexer Implementations

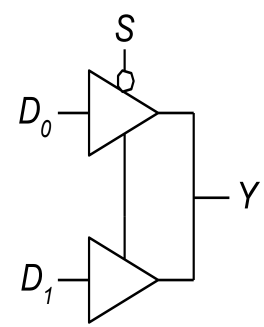
- **Logic gates**
  - Sum-of-products form



$Y = D_0\overline{S} + D_1S$



- **Tristates**
  - For an N-input mux, use N tristates
  - Turn on exactly one to select the appropriate input

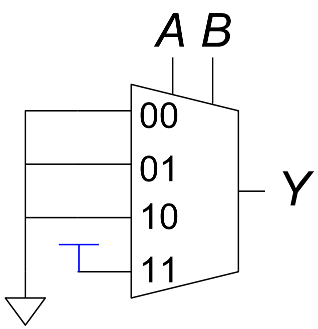


Logic using Multiplexers

- Using the mux as a lookup table

A	B	Y
0	0	0
0	1	0
1	0	0
1	1	1

$$Y = AB$$



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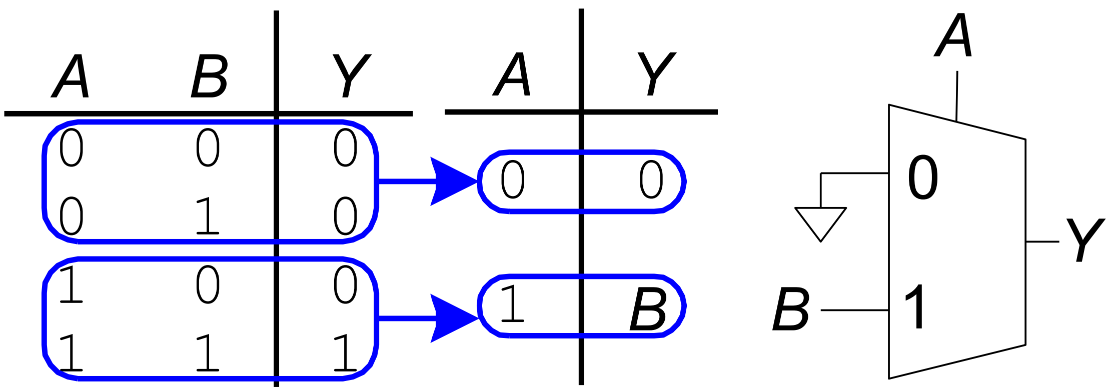
Logic using Multiplexers

- Reducing the size of the mux

$Y = AB$



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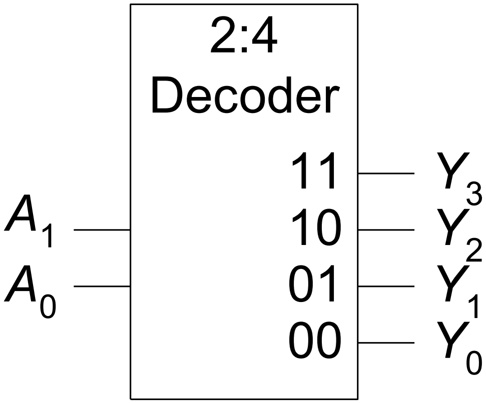
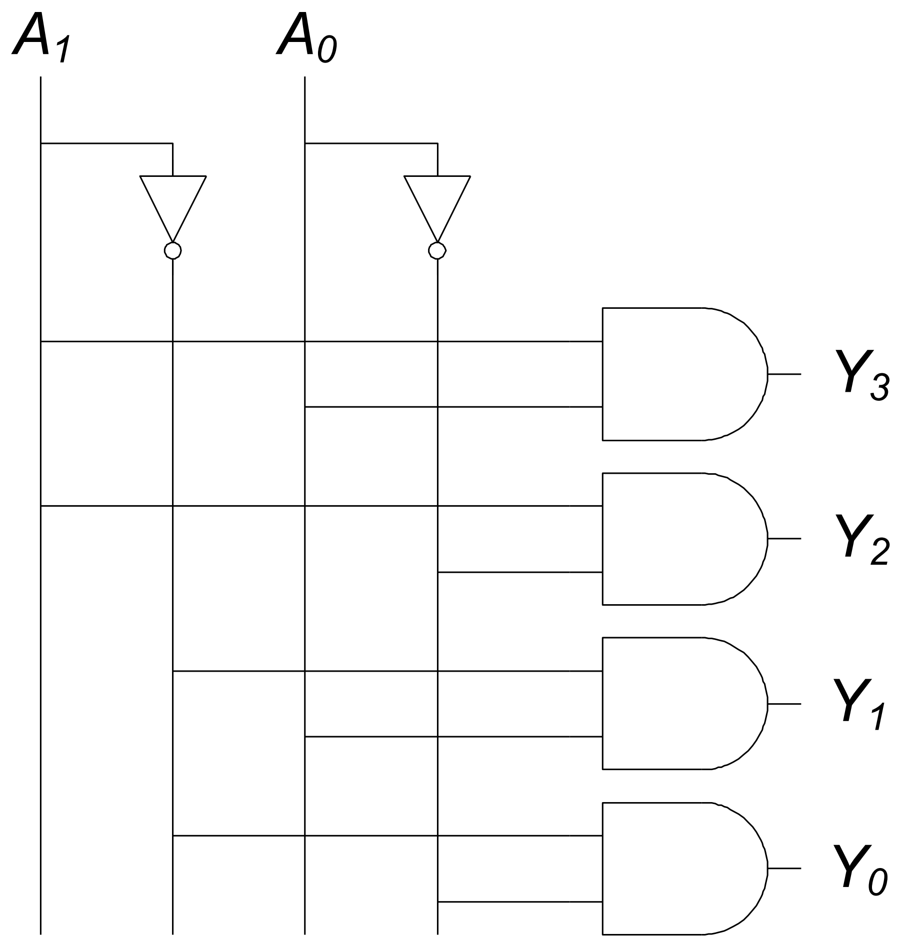
Decoders

- N inputs, 2^N outputs
- One-hot outputs: only one output HIGH at once



A_1	A_0	Y_3	Y_2	Y_1	Y_0
0	0	0	0	0	1
0	1	0	0	1	0
1	0	0	1	0	0
1	1	1	0	0	0

Decoder Implementation



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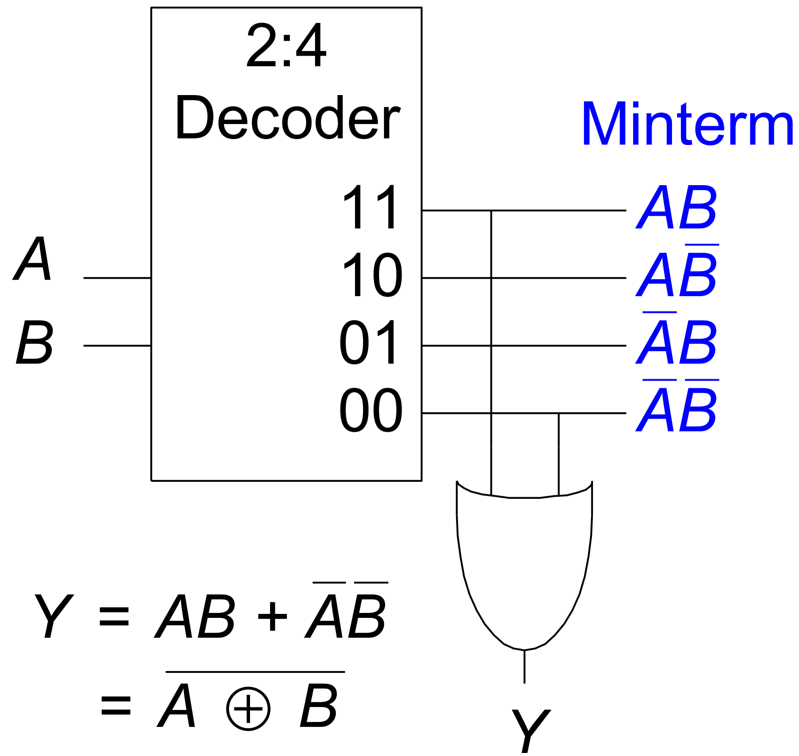
FROM ZERO TO ONE

Logic Using Decoders

- OR minterms



$$Y = AB + \overline{A}\,\overline{B}$$
$$= \overline{A \oplus B}$$

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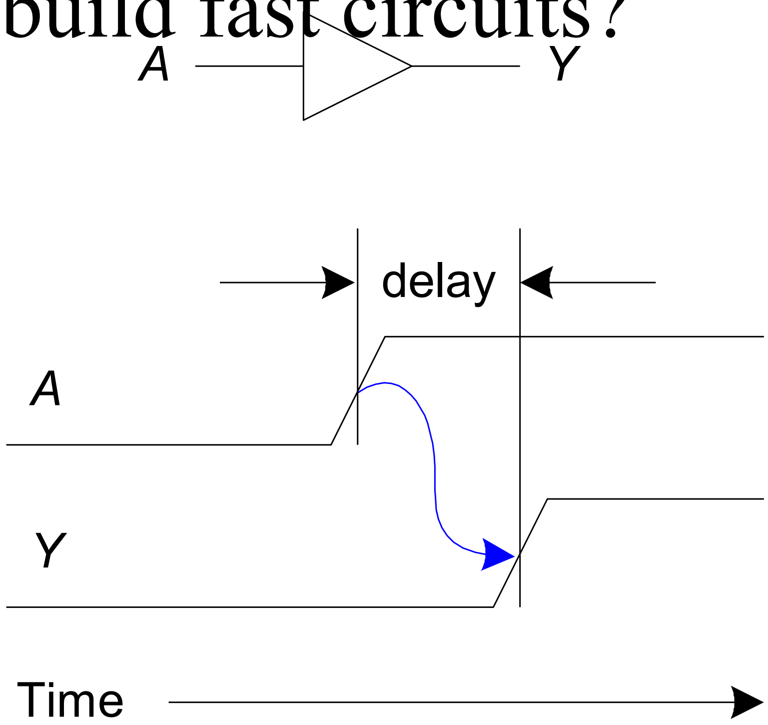
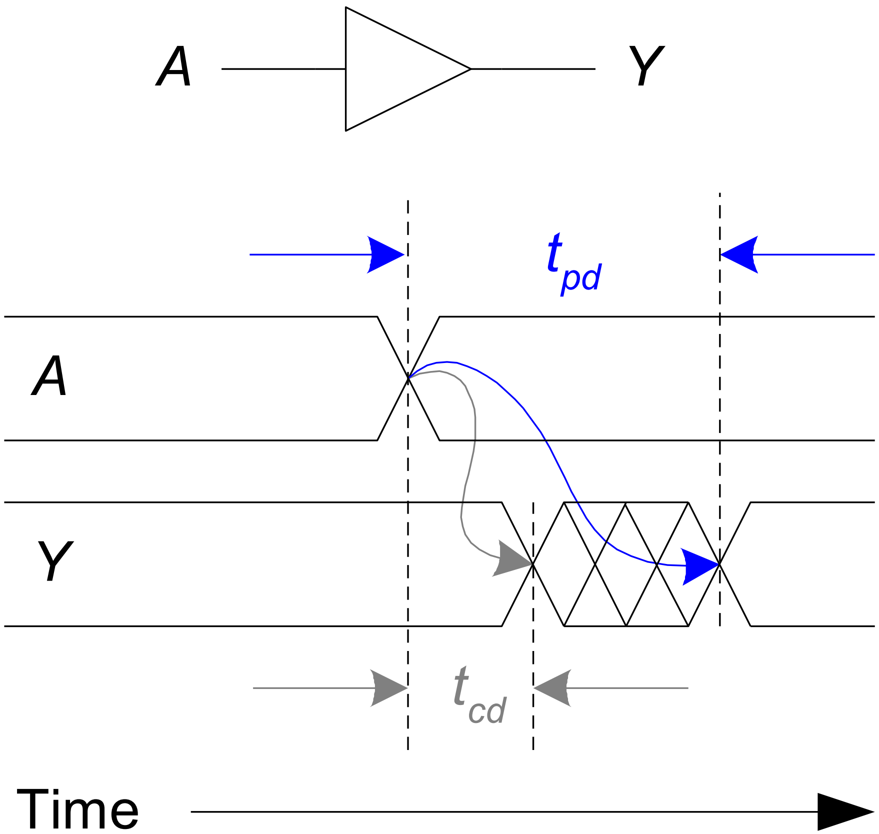
Timing

- Delay between input change and output changing
- How to build fast circuits?



Propagation & Contamination Delay

- **Propagation delay:** t_{pd} = max delay from input to output
- **Contamination delay:** t_{cd} = min delay from input to output



FROM ZERO TO ONE

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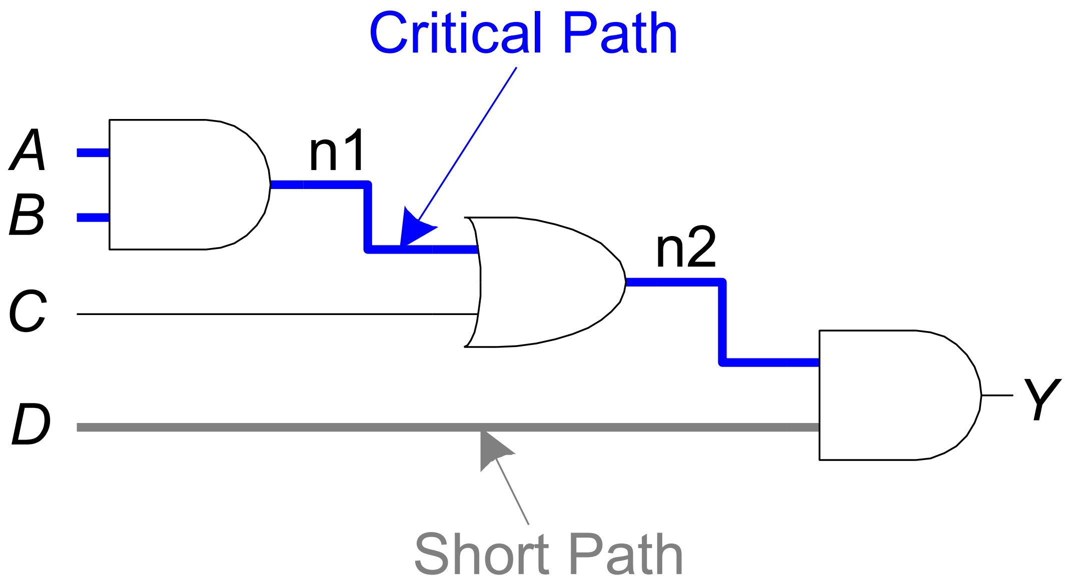
Propagation & Contamination Delay

- Delay is caused by
  - Capacitance and resistance in a circuit
  - Speed of light limitation
- Reasons why t_{pd} and t_{cd} may be different:
  - Different rising and falling delays
  - Multiple inputs and outputs, some of which are faster than others
  - Circuits slow down when hot and speed up when cold

Critical (Long) & Short Paths



Critical (Long) Path: $t_{pd} = 2t_{pd\_AND} + t_{pd\_OR}$

Short Path: $t_{cd} = t_{cd\_AND}$

FROM ZERO TO ONE

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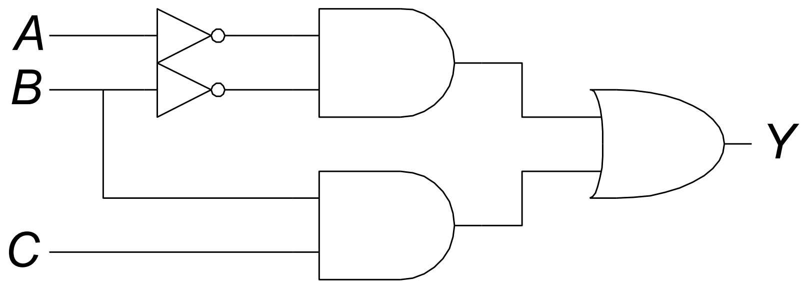
Glitches

- When a single input change causes multiple output changes

Glitch Example

- What happens when A = 0, C = 1, B falls?



Y (C \ AB)	00	01	11	10
0	1	0	0	0
1	1	1	1	0

$$Y = \overline{A}\,\overline{B} + BC$$

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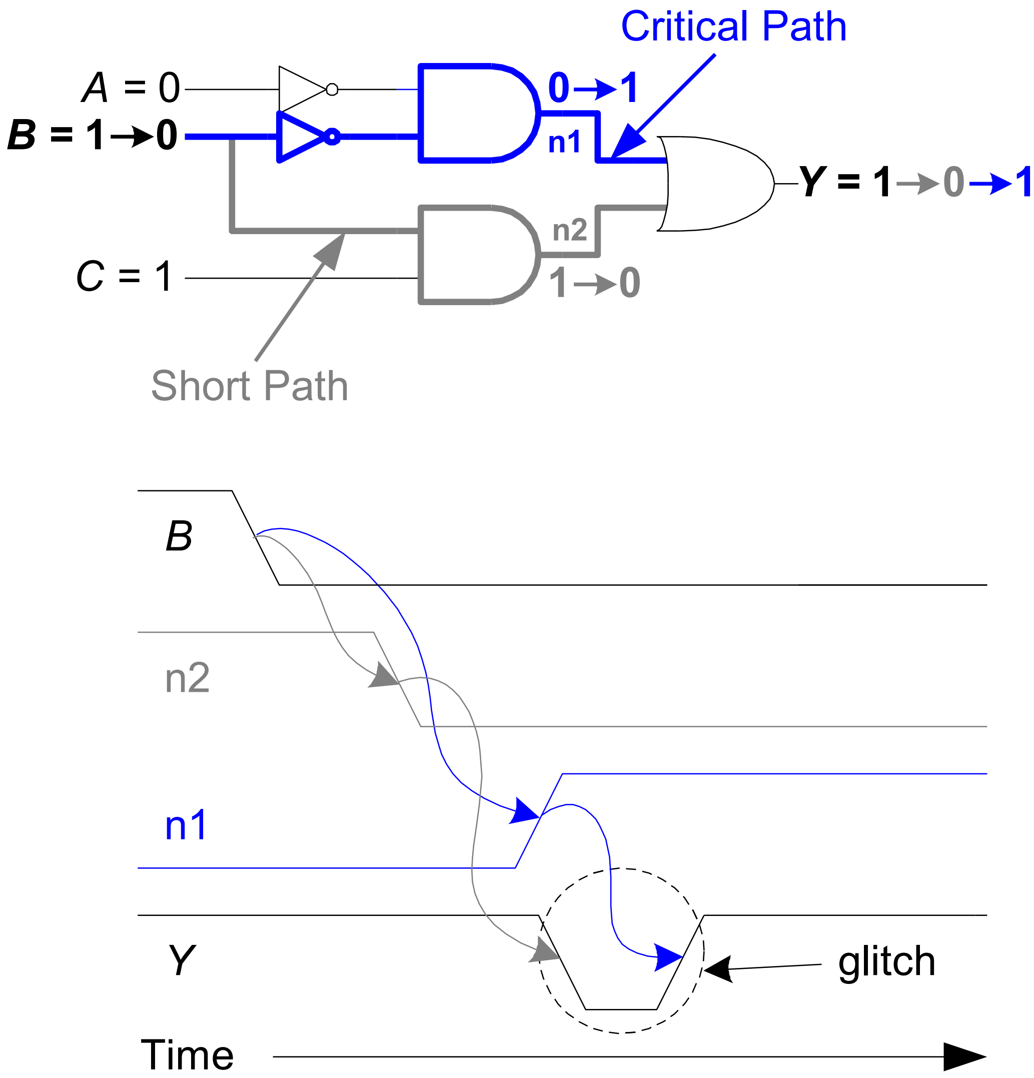
Glitch Example (cont.)



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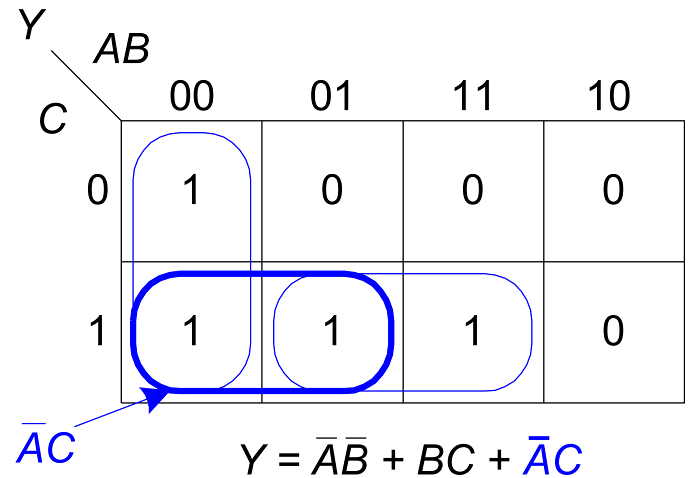
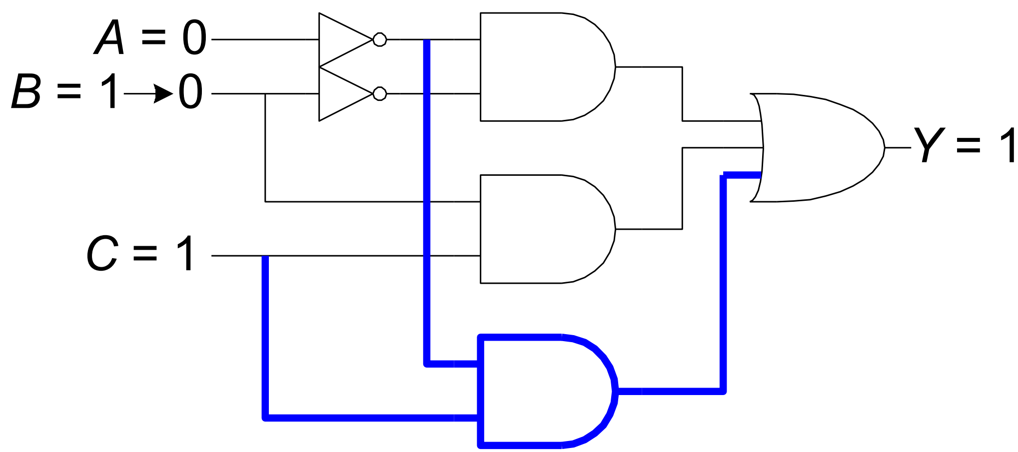
Fixing the Glitch



$Y = \bar{A}\bar{B} + BC + \bar{A}C$



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Why Understand Glitches?

- Glitches don't cause problems because of **synchronous design** conventions (see Chapter 3)
- It's important to **recognize** a glitch: in simulations or on oscilloscope
- Can't get rid of all glitches – simultaneous transitions on multiple inputs can also cause glitches