Lecture 6

3D plots and animation

1 Introduction

The majority of graphing tasks we face are typically two-dimensional functions of the form $y=f(x)$. However, not all functions have a single "input" and a single "output." The motion of a particle through space is described by vector position vs time

$$\mathbf{r}(t)=[x(t),y(t),z(t)]$$

We could represent this by three 2D plots, but a more physical representation would be to trace the particle trajectory in a single 3D plot. In this case the independent variable t does not form one of the plot axes. Instead it is a *parameter* of the motion. The resulting graph is called a *parametric plot*.

Some engineering problems deal with *fields*. A field is physical property which can vary throughout space. For example, the variation of ground elevation across a region of Earth's surface can be expressed as

$$z=f(x,y)$$

Here the two coordinates x,y might correspond to longitude and latitude and z to ground elevation, possibly obtained from surveying. Or, z might represent surface temperature or atmospheric pressure. In those cases we might also be interested with variation through time as well as through space.

Since a computer screen is two dimensional, plots in three (and higher) dimensions will necessarily have to represent a single *projection* of the function. Different projections might highlight certain aspects of the function and obscure others. This problem grows with the number of dimensions and is why *scientific visualization* is an active field of research.

In this lecture we want learn a few basic 3D plotting techniques. We will use the following code

```
Nx = 80;
Ny = 40;
x = linspace(-6,6,Nx);
y = linspace(-3,3,Ny);
z = zeros(Ny,Nx);
for i=1:Nx
  for j=1:Ny
    z(j,i) = cos(x(i))*sin(y(j));
  end
end
```

to generate an array of z values which we will plot in various ways. Notice that the first index of the z array corresponds to the y coordinate and the second index to the x coordinate. This relates to the *raster scan format* traditionally used on computer monitors and the way arrays appear in graphics cards. Both Scilab and Matlab use this convention.

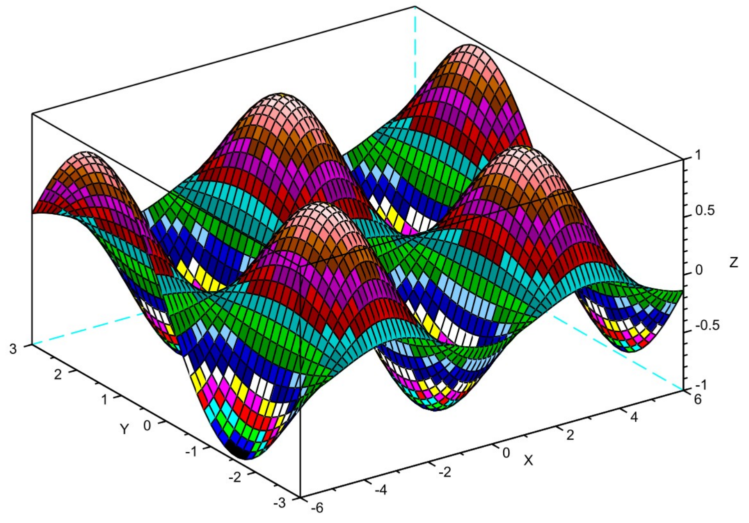


Fig. 1: Output of surf(x,y,z) command

2 The surf and mesh commands

Given any 2D array of real numbers *z* either of the commands

```
surf(z);
mesh(z);
```

will plot the *z* values as the elevation field of a 3D surface. The mesh command shows this surface as a "wire mesh" while the surf command show it as a solid color-coded surface. The *x* and *y* coordinates are the integer indicies of the array. Alternately we can explicitly provide *x* and *y* values

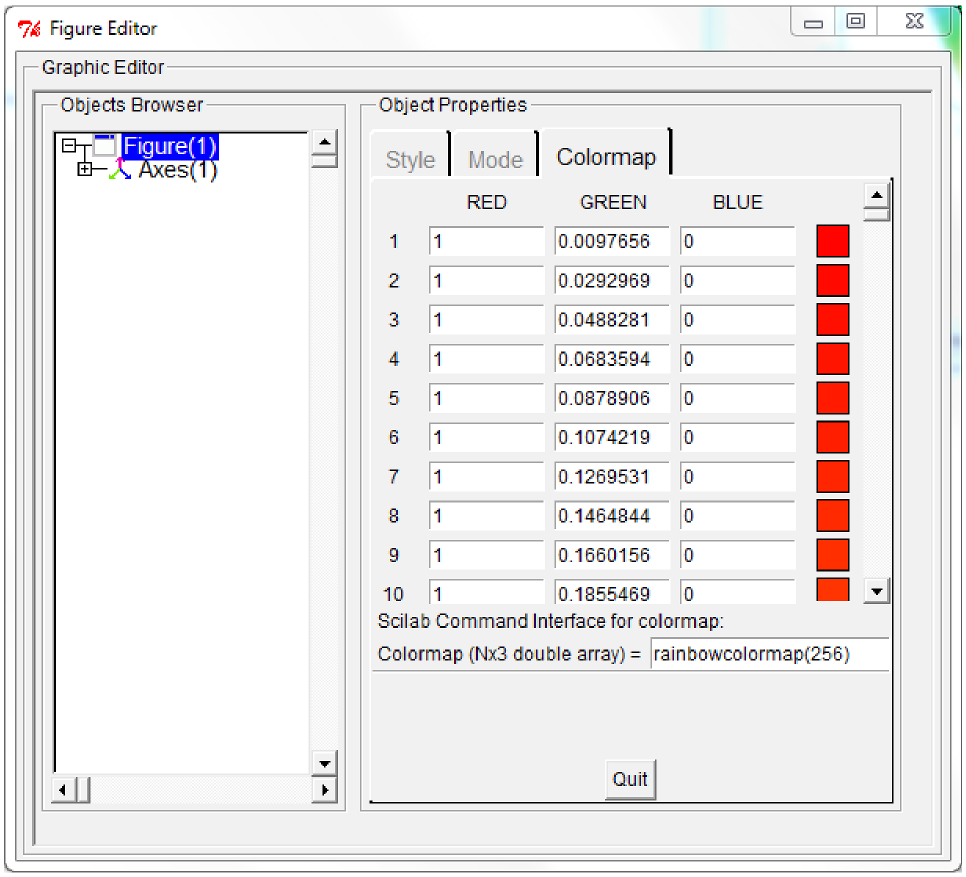
```
surf(x,y,z);
```

The result for our data is shown in Fig. 3. Because this is a 2D projection of a 3D surface, some parts of the surface may be obscured. To get different views use the Rotation tool from the Tools menu. Click (right button in Scilab, left button in Matlab) and drag with your mouse to reorient the surface. As in the 2D case, we can export a figure to a graphics file for inclusion in a presentation or paper.

2.1 Changing figure and axes properties interactively (Scilab)

As with 2D plots we can use the Edit => Figure properties menu option to change the appearance of our figure. Similar options are available in Scilab and Matlab but the details differ. We will consider Scilab.

One of the most visually noticeable changes we can make is to use a different *color map*. The color map specifies how different *z* values are mapped into different colors. This can be changed by either directly editing the red-green-blue values (Fig. 2) or more practically by specifying a color map in the Colormap dialog box. Scilab has several predefined color maps (see help colormap).



	RED	GREEN	BLUE	
1	1	0.0097656	0	🟥
2	1	0.0292969	0	🟥
3	1	0.0488281	0	🟥
4	1	0.0683594	0	🟥
5	1	0.0878906	0	🟥
6	1	0.1074219	0	🟥
7	1	0.1269531	0	🟥
8	1	0.1464844	0	🟥
9	1	0.1660156	0	🟥
10	1	0.1855469	0	🟥

Scilab Command Interface for colormap:

Colormap (Nx3 double array) = rainbowcolormap(256)

Fig. 2: Changing the color map.

Two of the most useful are

```
rainbowcolormap(n)
jetcolormap(n)
```

The integer `n` determines the number of discrete colors. A larger number, such as 256 gives a smooth variation of color throughout the figure. But, you may want to have only eight discrete colors in which case use 8 as the argument.

Using the Axes Editor we can change axis labels, figure title and the numerical label font for the *x*, *y* and also *z* axes. In the Aspect submenu deselecting Cube scaling will produce a more geometrically accurate representation of the surface. By also selecting the Isoview option the surface plot will correspond to a physical representation of the surface with equal scaling for the *x*,*y*,*z* axes.

Another useful command is

```
colorbar(zmin,zmax);
```

which adds a color bar to the figure showing the relationship between color and *z* value. After various formatting changes our figure appears as shown in Fig. 4.

2.2 Changing figure and axes properties within a program (Scilab)

As with 2D plots we can execute commands within a program to make formatting changes.

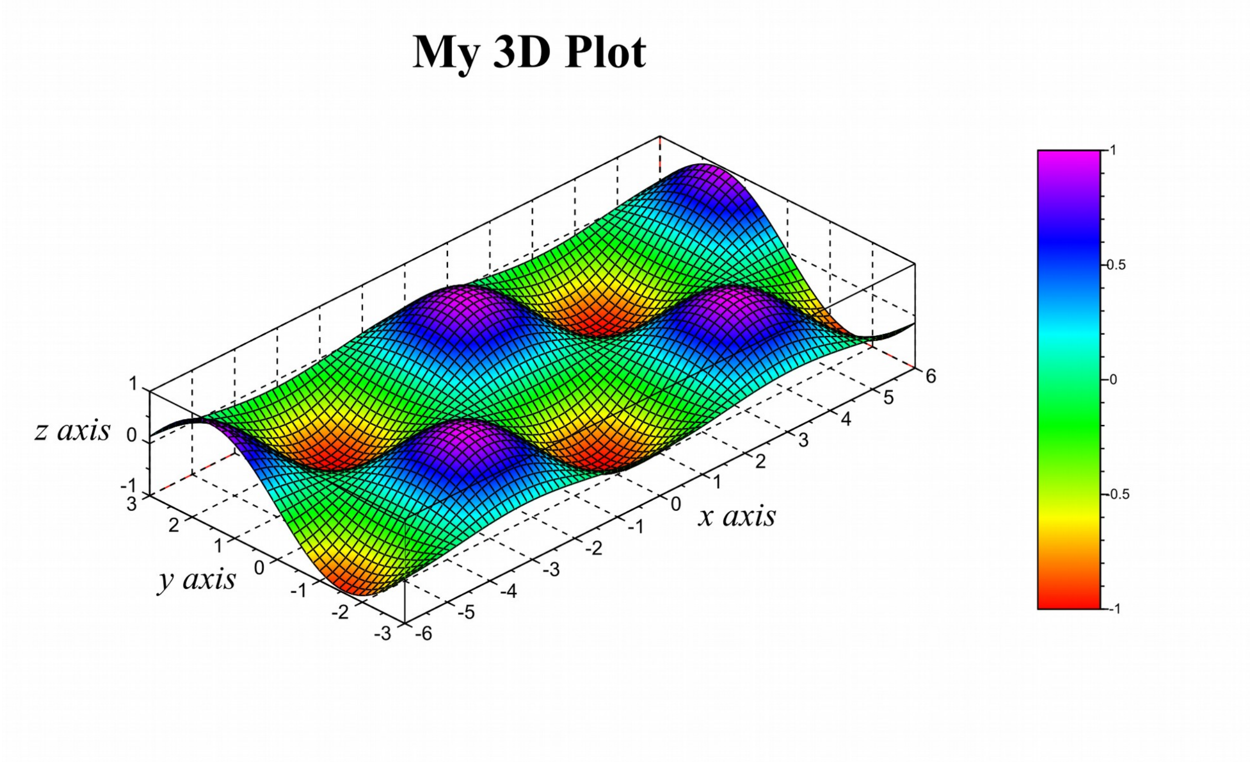


Fig. 3: Plot after applying format changes

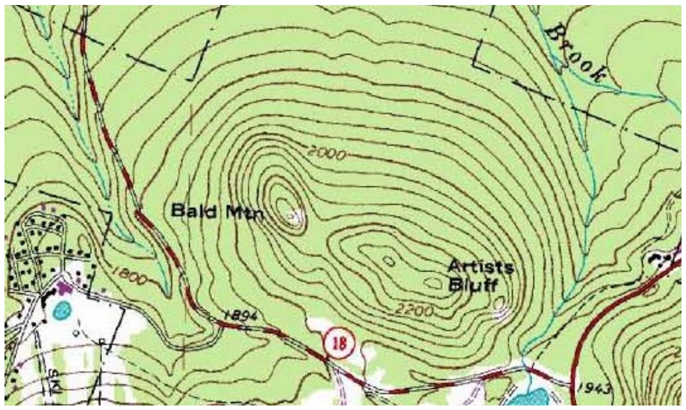
Fig. 4 was generated by the following commands.

```
surf(x,y,z);
h = gce();
h.color_flag = 3;

fg = gcf();
fg.color_map = rainbowcolormap(256);
fg.figure_size = [900,600];

ax = gca();
ax.cube_scaling = "off";
ax.isoview = "on";
ax.font_style = 1;
ax.font_size = 3;
ax.grid = [0,0,0];

ax.x_label.text = "x axis";
ax.x_label.font_style = 3;
ax.x_label.font_size = 5;
ax.y_label.text = "y axis";
ax.y_label.font_style = 3;
ax.y_label.font_size = 5;
ax.z_label.text = "z axis";
ax.z_label.font_style = 3;
ax.z_label.font_size = 5;

ax.title.text = "My 3D Plot";
ax.title.font_style = 4;
ax.title.font_size = 6;

ax.rotation_angles = [60,-45];
ax.data_bounds = [-6,-3,-1;6,3,1];

colorbar(-1,1);
```

The get current entity command `gce()` and the `color_flag` subtly change the way the surface color is interpolated. You can experiment with flag values of 0 through 4. If you use these commands they must come immediately after the `surf` plotting command.

3 The contour command

Another way to represent surface elevation is by drawing labeled contours of constant elevation on an *x,y* representation of the surface as is done in a topographic map.



```
contour(x,y,z,n);
```

Here `n` is the number of (uniformly spaced) contour levels you want drawn on the figure. Replacing `contour` by `contourf` creates a filled contour plot. One irritation in Scilab is that if we following the "raster scan" format we used with our initial data-generating code, we have to replace the `z` argument with its transpose `z.'` (this is not the case in Matlab). As with all graphics, we can adjust the formatting to our liking to get something such as shown in Fig. 4.

For the plain `contour` command Scilab adds numerical labels to the contours by default. I find these to be too messy to be of much use and prefer a color bar as shown in the figure. To turn off labeling use the `xset('fpf',' ')` command before the plotting, as shown below.

```
fg = scf(0);
clf();
fg.figure_size = [800,400];
fg.color_map = jetcolormap(11);
xset('fpf',' ');
contourf(x,y,z.',9);
ax = gca();
ax.isoview = "on";
ax.auto_ticks = ["on","on","on"];
ax.font_style = 3;
ax.font_size = 4;
ax.x_label.text = "longitude";
ax.x_label.font_size = 4;
ax.y_label.text = "latitude";
ax.y_label.font_size = 4;
ax.title.text = "ground elevation";
ax.title.font_size = 6;
colorbar(min(z),max(z));
```

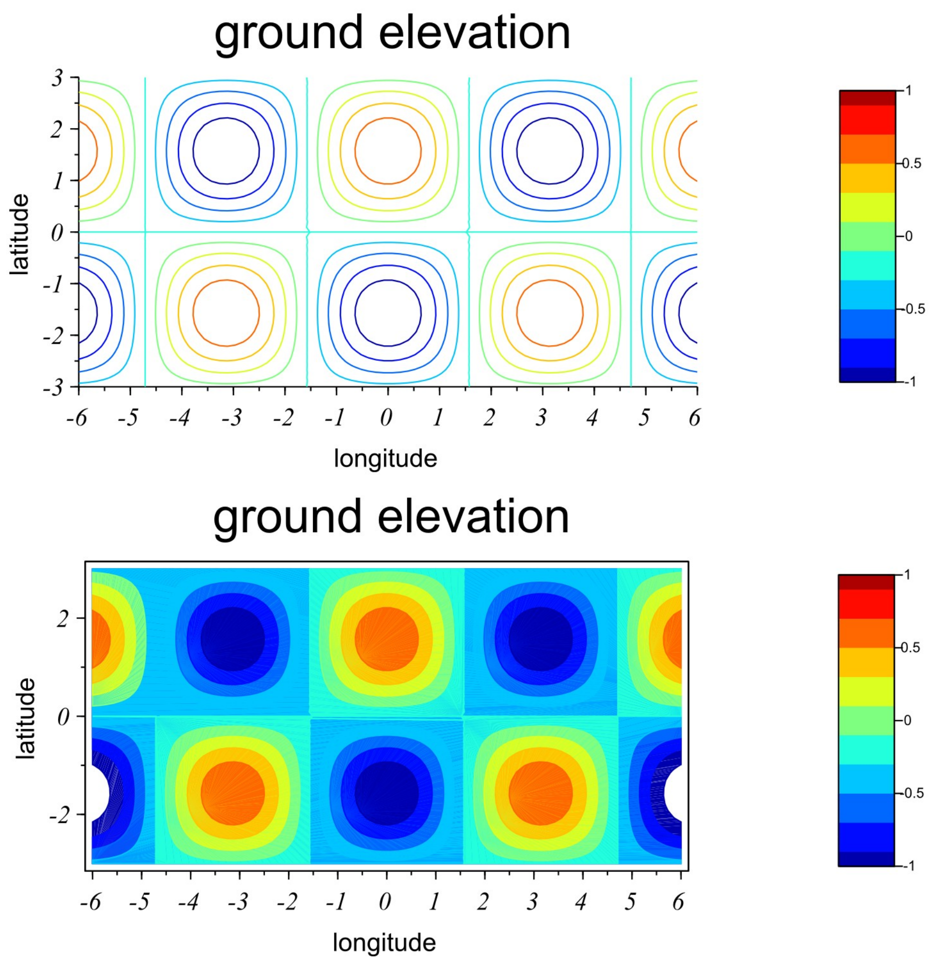


Fig. 4: Output of the contour and countourf commands with added formatting

4 Parametric plots

"Trajectory" plots of the form $[x(t), y(t), z(t)]$ can be generated by the commands

```
param3d(x,y,z); //Scilab
plot3(x,y,z); %Matlab
```

As an example, below is Scilab code to generate a spiral trajectory starting at the origin and extending up along the *z* direction as time increases. After some interactive orientation and formatting we end up with the graph of Fig. 5.

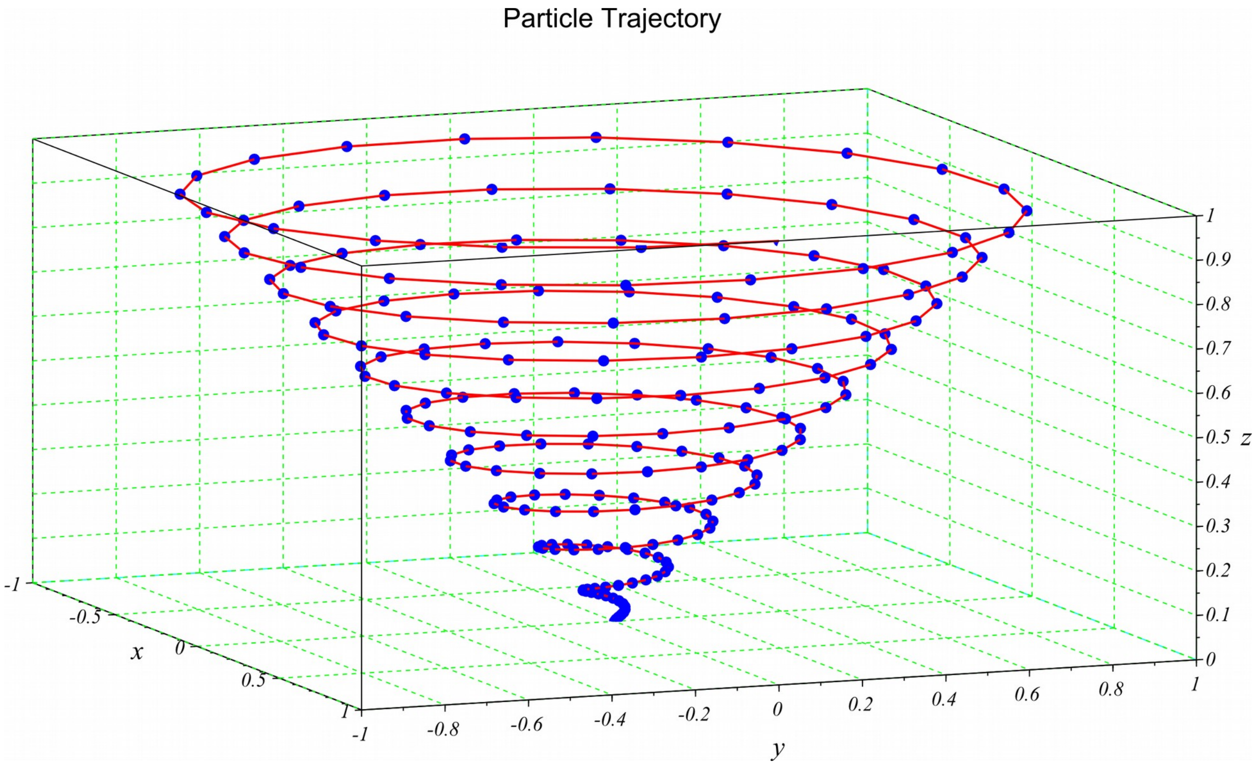


Fig. 5: Parametric "trajectory" plot using the param3d(x,y,z) command

```
Nt = 200;
t = linspace(0,10,Nt);
x = zeros(Nt,1);
y = zeros(Nt,1);
z = zeros(Nt,1);
for i=1:Nt
  r = t(i)/10;
  x(i) = r*cos(2*%pi*t(i));
  y(i) = r*sin(2*%pi*t(i));
  z(i) = r;
end
param3d(x,y,z);
```

5 Animation (Scilab)

Animation is simply the process of generating a series of graphic figures one for each frame of the animation. The frames can be displayed in "real time" on the screen or saved as graphics files which can later be assembled into a video file. There are a few subtle points which arise when generating animations. Let's illustrate by an example. We start with the following code

```
x = linspace(0,2*%pi,100);
y = sin(x);
nFrames = 200;
t = linspace(0,4*%pi,nFrames);
```

This is going to represent a vibrating string. Let's try to generate an animation as follows.

```
for i=1:nFrames
  plot(x,y*cos(t(i)));
end
```

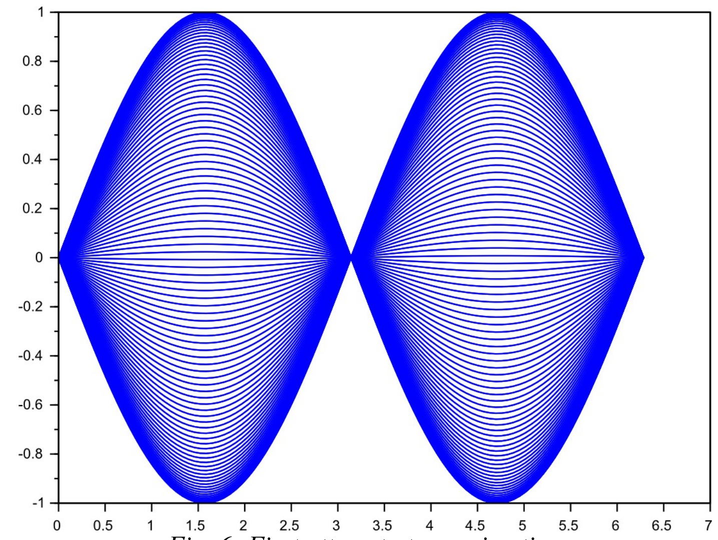


Fig. 6: First attempt at an animation

As we should have guessed, this plots one after another position of the string on the same plot producing the result in Fig. 6. We need to erase the old curve before plotting a new one. So we try

```
for i=1:nFrames
  clf();
  plot(x,y*cos(t(i)));
end
```

This produces a blinking mess in which the *y* axis is constantly rescaling. The rescaling we can get rid of by explicitly stating the data bounds

```
for i=1:nFrames
  clf();
  plot(x,y*cos(t(i)));
  ax = gca();
  ax.data_bounds = [0,-1;2*%pi,1];
end
```

The problem is that the screen is still a blinking mess. What is happening is that when we tell Scilab to clear the figure, we see it go blank. Then when we tell Scilab to draw a new figure, we see that appear. The result is irritating on/off "video flicker." What we really want is that as we are viewing one frame we are generating a new frame "behind the scenes." When we are ready for it we want the new frame to swap out the old frame "instantly." This requires two segments of memory or "video buffers." One holds the currently visible frame. The other "background" buffer is where the computer is generating the next frame. When ready, the computer rapidly copies the background buffer contents into the visible buffer. In video systems this process is called *double buffering*. Triple buffering is used in high-end video (e.g., gaming systems) so that while the buffer copying is occurring the computer can already be working on another frame. Scilab provides two commands to implement double buffering: `drawlater()` and `drawnow()`. They are very simple to use as shown in the following code.

```
for i=1:nFrames
  drawlater(); //turn on double buffering so that operations
  clf();       //occur in the background
  plot(x,y*cos(t(i)));
  ax = gca();
  ax.data_bounds = [0,-1;2*%pi,1];
  drawnow(); //copy the background buffer to the visible buffer
end
```

This solves our problems. Inbetween the `drawlater()` and `drawnow()` commands we can modify the figure in anyway we wish – adding labels, titles, changing color maps and so on.

The speed with which frames update depends on how long it takes to generate a new frame. If the goal is to produce an independent video file then we want to save each frame to disk. One approach is shown here.

```
scf(0);
for i=1:nFrames
  drawlater();

  //generate a new frame here

  drawnow();
  fname = msprintf("frames/f%03d.png",i);
  while (~isfile(fname))
    xs2png(0,fname);
  end
end
```

First we create a subdirectory names “frames” before running the animation code. During the animation rendering the `msprintf` function creates a series of file names from the frame index `i`. If this png file does not already exist it is written from the current graphics frame. The result is a sequence of png files

```
f001.png , f002.png , ...
```

in the subdirectory frames. From there video editing software can be used to produce an animation file. A useful free and open-source program for this is Virtualdub (virtualdub.org).