# A Simple Algebraic Approach to Teaching Oligopoly Models

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# A Simple Algebraic Approach to Teaching Oligopoly Models

In this paper I develop a simple algebraic model which provides a new perspective on (1) how we teach the interaction between firms in a Stackelberg model and (2) how the same framework can be extended to a discussion on entry into an industry. Besides providing a different approach to teaching the Stackelberg model, my framework has the additional advantage that it is intuitive, technically simple and thus readily accessible to students who may not have a background in calculus.

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## Section 1: Introduction

Any undergraduate course in Intermediate Microeconomics or Industrial Organization devotes a considerable amount of time to discussing the Stackelberg model of oligopoly. In this paper I develop a simple algebraic model which provides a new perspective on

(1) how we teach the interaction between firms in a Stackelberg model

(2) and how the same framework can be extended to a discussion on entry (such as the Dixit-Spence model) into an industry

Most undergraduate texts, Carlton and Perloff (1994), Mansfield (1997), Varian (1996), Schotter (1997), Eaton and Eaton (1995), Salvatore (1997) among others, present simple and lucid analyses of the first topic while the second is usually reserved for graduate texts in economics.

The current paper is not intended to supplant the exposition provided in contemporary textbooks. It does however intend to inform and improve undergraduate teaching in microeconomics and industrial organization by providing a different and intuitively appealing way of explaining the Stackelberg model. Moreover the paradigm that I develop can be extended to a general discussion of different modes of entry into a market. Discussion on entry can be motivated by pointing out that institutional factors or government regulations mandating information disclosure may dictate whether the competition between firms is simultaneous or sequential. For instance one firm may have a patent that allows the firm to enter the industry first and a second firm enters the industry after the patent expires. Economists have rationalized quantity competition in this context to imply physical capacity – for instance each firm decides on the size of the physical plant that they wish to set up. Establishing capacity and expanding it is costly so firms want to make sure that they are establishing the optimum capacity. In Cournot's story both firms enter the market simultaneously and hence have to decide on how much capacity to install without knowing the other firm's choice. In Stackelberg, however, they make the decision sequentially – one firm moves first and the second firm observes the first firm's choice before choosing its own capacity. I discuss these issues in greater detail in Section 2.3.

The value added of the paper comes from two different sources. (1) It develops a framework that provides a different way of explaining and a simple mathematical solution to the Stackelberg model. The same framework also allows a simple and lucid discussion of entry into an industry – a topic that usually requires a fair amount of technical sophistication. This treatment of entry is not found in any of the current undergraduate texts. (2) The paper achieves this within a simple algebraic framework that will be accessible to undergraduates who may not necessarily have a strong background in calculus.

Section 2 sets up the model. Section 2.1 provides a discussion of the standard Cournot model as a way of motivating the issues later in the paper. Section 2.2 discusses a new approach to the Stackelberg model. Section 3.1 shows how the new approach developed in Section 2.2 can be extended to a discussion of the Dixit-Spence model of entry deterrence. Section 3.2 shows how different modes of entry can be handled within the same framework. Section 4 provides an overview of where this material fits in the overall scheme of things in the course of a standard course in intermediate microeconomics or industrial organization. Finally section 5 concludes.

# Section 2: The Model

## 2.1: Cournot Model of Oligopoly

Consider a market with 2 firms that produce a homogeneous product. The inverse demand function is given by P = A – Q where $Q = Q_1 + Q_2$. Assume the firms to be identical in the sense that they both have a marginal cost of $c per unit. The two firms compete in quantities as Cournot duopolists. The solution to this problem is simple. See Carlton and Perloff (Second Edition, 1994, pp. 233-238) for a simple algebraic technique for deriving the best response function for the two firms. It is not difficult to demonstrate to the students that the best response function for firm 1 is given by $Q_1 = (A-Q_2-c)/2$ and that for firm 2 is $Q_2 = (A-Q_1-c)/2$. After that one can show how this is simply a simultaneous equation system involving two equations in two variables which can be solved for $Q_1$ and $Q_2$ obtaining the values $Q_1^* = (A-c)/3$ and $Q_2^* = (A-c)/3$.

*For the sake of simplicity and without loss of generality set A=1 and c=0.* So the demand curve becomes P = 1- Q and the marginal cost of production is zero. In this scenario each Cournot duopolist will produce an output of 1/3 each. Total output in the market is Q=2/3. The market price is P=1/3. Each firm makes a profit equal to 1/9.

## 2.2: The Stackelberg Model

Next I turn to the Stackelberg model. Most textbooks present an adequate explanation of the model. For a rigorous analysis of the model see Calton and Perloff, 1994, pp. 250-252.

At this point I will point out the informational asymmetries in the Cournot and Stackelberg model. Cournot is a game involving simultaneous moves. In the Cournot model each firm moves simultaneously and does not know the output decision of the other firm. Each firm has to decide on its output (capacity) *without knowing what the other firm is doing*. In a Stackelberg model on the other hand each firm knows *what the other firm is doing*. Firm 1, when it moves, knows exactly how firm 2 will respond to its output (capacity) since firm 1 knows that firm 2's best response function is $Q_2=(A-Q_1-c)/2$. Firm 2 in turn when it decides its output knows exactly what output firm 1 has produced.

The usual discussion of the Stackelberg model starts by introducing the concepts of backward induction and subgame perfection. Then we show students how to solve firm 2's profit maximization exercise and obtain its reaction function $Q_2=(A-Q_1-c)/2$. Then we work our way back and solve firm 1's profit maximization exercise by plugging in firm 2's reaction function into firm 1's profit function. We then solve for the two firms' quantities in the usual way.

But there is a simpler way of showing this, assuming that the cost function is linear[1]. Let us continue with the same numerical example where A=1 and c=0, i.e. firms face a linear demand curve of the form P=1-Q and produce at zero marginal cost.

I simply point out that when firm 1 makes its output decision it is in fact acting as a monopolist since there are no other firms in the market. So facing a demand curve of the form P=1-Q, this firm goes ahead and produces the monopoly output which is ½. (Since discussions of oligopoly models succeed monopoly students by this time understand why this is the monopoly output.) Once it has done so how much of the market is left? Exactly ½. What does firm 2 do when it is his turn to move? He is really

[1] It should be noted that the results derived in the following pages about the Stackelberg and Dixit-Spence models are based on the assumption of a linear cost function and may not work for non-linear costs. But given that in both undergraduate, as well as graduate, classes we exclusively use linear cost functions to

looking at the residual demand in the market which is of the form P=1/2-Q. But now firm 2 is in effect a monopoly in this residual market and so it goes ahead and produces the corresponding monopoly output which is ¼. It is easy to check that solving the Stackelberg model in the usual way would yield $Q_1$=1/2 and $Q_2$=1/4 as the output response of firm 1 and firm 2 respectively. The resulting market price is P=1/4. This price of ¼ is lower than the Cournot price of 1/3. Firm 1 makes a profit of 1/8 while firm 2 makes a profit of 1/16. Firm 1 is then much better off than being a Cournot competitor while firm 2 is worse off with a lower level of profit. In Cournot both firms get a profit of 1/9.

What if there are more than two firms moving sequentially? Consider three firms. In that case firm 1 moves first and produces ½; firm 2 moves next and produces 1/4. How much of the market is left? ¼. So firm 3 in turn chooses the monopoly output in the residual part of the market which is 1/8. The aggregate output is 7/8. The market price is 1/8. Firm 1 makes a profit of 1/16, firm 2, 1/32 and firm 3, 1/64. Again it is easy to check that these are indeed the three outputs that would prevail if we solve the model in the usual way. If the three firms competed as Cournot oligopolists then each firm would have produced an output of ¼. The market price would have been higher at ¼. Each firm would have enjoyed a profit of 1/16. So firm 1 is better off as the Stackelberg leader than as a Cournot competitor while firms 2 and 3 are worse off.

### 3.1 The Dixit-Spence Model of Entry Deterrence

The Stackelberg model of the previous section can be used as a point of departure to introduce the idea of entry deterrence. Obviously merely moving first and producing

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motivate these models, this assumption is justified. Also this assumption allows us to extend the discussion to a number of issues that would, usually, be far outside the scope of a typical undergraduate course.

the monopoly out put is not enough to deter entry since there is enough residual demand for a follower to enter the market and make profit. So the question arises – what should an incumbent firm do in order to prevent other firms from entering the market?

Usually the first model that is introduced is the Bain-Modigliani-Sylos-Labini model. The model has three periods. In period 1 the incumbent firm chooses an output and commits to maintaining this output even after entry has occurred. In period 2 the potential entrant decides to enter or not. Then in period 3 the incumbent firm chooses its output. According to the BMS-L model this output is the same as the one that the incumbent firm had committed to before. However as we know the BMS-L model is not subgame perfect since *once entry has occurred it is not profitable to produce the pre-entry output but rather accommodate entry and behave as a Cournot duopolist.* See Schotter (1996, pp. 409-11) for a lucid analysis of this model. The threat of expanding output to the point where entry is not profitable for the entrant and maintaining that output even if entry occurs is a non-credible threat.

So what would be a credible threat? At this point we can introduce the Dixit-Spence model. As we know in the Dixit-Spence model firms should over-invest in capacity and install much more capacity than they would in the standard Cournot or Stackelberg duopoly story. Installing this excess capacity and then threatening to expand output following entry is a subgame perfect equilibrium.

In the Dixit-Spence model the incumbent decides in period 1 as to how much capacity to install. Then in period 2, the incumbent firm chooses its output, followed by the potential entrant's decision to enter or not. The incumbent firm then can respond by adjusting its output in period 3.

Let us assume at this point that the incumbent firm has the option of installing a certain amount of capacity K*, such that for any output less than K*, the incumbent firm can produce at zero marginal cost as before. But for any quantity greater than K*, the cost per unit becomes ¼. For the potential entrant the cost per unit is ¼ since the entrant has no installed capacity. So the entrant's cost function is

$C = 0$ for $q \leq K^*$

$C = (1/4)q$ for $q > K^*$.

Let us continue with our simple demand of $P = 1 - Q$. The first question is – what is the output that would make entry unprofitable? The answer is simple – set price equal to average cost. At this point the incumbent firm breaks even and any potential entrant is faced with non-positive profit. Since c is 1/4, then setting price equal to average cost yields a capacity of 3/4 unit. The incumbent firm should install capacity of 3/4 unit but produce its monopoly output of ½. Faced with this installed capacity of ¾ unit, entry becomes unprofitable since if the incumbent firm does produce ¾ then the residual market demand is $P = 1/4\text{-}Q$ and even the monopoly output in this residual market is 0. The incumbent firm can now costlessly expand output till 3/4 unit making such entry unprofitable. Thus in this model the incumbent firm installs just enough capacity that will make entry unprofitable and then in the equilibrium of this game, entry does not occur and the incumbent firm continues to enjoy monopoly profit.

If the incumbent does not have installed capacity then it cannot deter entry by a potential entrant. In this case produces its optimal output as the Stackelberg leader which would be 3/8 and that of the follower would be 3/16.

### 3.2 Handling Various Modes of Entry

I can also talk about a wide variety of entry sequences without any difficulty. How to motivate this idea of different modes of entry? Aoki and Prusa (1997) point out that institutional factors and government regulations that mandate information disclosure may determine whether competition takes the form of a simultaneous move game or a sequential move game. One good example is the differences in the timing of information disclosure between the U.S. and Japanese patent systems. Under the Japanese system of *kokai* a firm's patent application is laid open 18 months after the filing date and *before it is granted to the firm.* Hence it is possible for a firm to apply for a patent knowing the exact specifications of a rival firm's patent application. In contrast, in the U.S. the only way a firm learns about a rival's innovation is upon the actual granting of the patent to the rival. See Aoki and Prusa (1996, 1997) for detailed discussions of the impact of patent laws on the mode of market entry.

In the U.S. context then a firm with a patent may enter an industry first to be followed by others when the patent expires. But in the Japanese context since the patent is thrown open to a firm's rivals before it is granted to a firm, it is not inconceivable to think of more than one firm entering a market simultaneously to be followed by others at a later date. Such differences in patent law could explain how there could be different entry sequences into a market.

I can now pose problems such as:

(1) Suppose firm 1 moves first and decides what output to produce; firms 2 and 3 observe firm 1's output and then move simultaneously. Calculate the output produced by each firm.

(2) Suppose firms 1 and 2 act as Cournot duopolists and decide on their output. Firm 3 observes the output of both firms 1 and 2 and then decides on its output. Calculate the output of each firm in this case.

For (1) I say the following. When firm 1 makes its output decision it acts as a monopoly and as a result faces the entire market demand P=1-Q and chooses the monopoly output of ½. How much of the market is left for firms 2 and 3? The remainder of the market. So firms 2 and 3 are facing a market demand curve of the form P=1/2-Q. What output do they each produce? They are Cournot duopolists in the market except instead of getting the entire market demand they get ½ of it. So they produce 1/3*1/2=1/6 each. Again solving the model using the usual technique of subgame perfection (of course using calculus) one gets the same exact answer.

Similarly, for (2) firms 1 and 2 are duopolists and will therefore produce 1/3 each accounting for 2*1/3=2/3 of the market. So firm 3 in effect faces a market demand of the form P=1/3-Q. When firm 3 makes its output decision it can act as a monopolist and hence chooses the monopoly output in the residual part of the market which is ½*1/3=1/6. Again it is easy to check that these are indeed the output that would be produced in the Stackelberg model.

## Section 4:

Most textbooks provide adequate coverage of models of monopoly as well as oligopoly models such as the Cournot or Stackleberg model. However most undergraduate texts in microeconomics (Schotter's textbook is an exception) do not address questions of entry and entry-deterrence, such as the Dixit-Spence model, in any great depth. The textbooks in industrial organization such as Carlton and Perloff and

Pepall et al do address the issue but the discussion is sufficiently involved and complicated putting it beyond the grasp of undergraduate students who are not familiar with calculus. (Many undergraduate business majors who have to take a course in intermediate microeconomics fall in this category). However a simple discussion of entry-deterrence adds enormously to lectures in this topic because students usually find the discussion topical, relevant and stimulating.

I typically introduce the above topics after I have thoroughly covered monopoly, monopolistic competition and given the students a rudimentary introduction to simultaneous move games and sequential move games. I make sure that the students understand my discussion of how a standard unregulated monopolist makes his decision regarding price and output. Since my subsequent discussion of oligopoly models is dependent on the students having a good grasp on this topic. Then I use simple games to highlight the relevant game theoretic concepts such as best responses and by this time I want my students have an intuitive understanding of the Prisoner's Dilemma, as well as the concepts of Nash Equilibrium and backward induction. It is after these topics that I introduce the Cournot model. Then I discuss the Stackleberg model. First I discuss the standard textbook approach and then I show the students the new method. At which point I can extend my lectures to topics that are usually never included in most undergraduate courses. I talk about the rationale and incentive for deterring entry to perpetuate a monopoly position which guarantees higher profit. Then I can also talk about different sequences of entry into the product market.

Part of the reason for introducing issues of entry deterrence was motivated by the way we teach our students about monopolists. Most of the monopolies that we can immediately think of are usually regulated “natural monopolies”, with constantly decreasing average cost, such as public utilities. However most microeconomics textbooks concentrate at great length on unregulated monopolies with the “standard” U-shaped average cost. But when it comes to providing examples of such unregulated monopolies we are at a little bit of a loss. The examples found most often in textbooks is De Beers, the diamond producer or another producer who has obtained a monopoly position due to their holding of a patent.

But it is important and instructive to point out to students that there is yet another reason why a firm may enjoy a monopoly position – because it has successfully prevented other firms from entering. But if one introduces that idea then that automatically necessitates talking about entry and entry-deterrence. The model most often referred to is the Dixit-Spence model where firms hold excess capacity to deter entry. As I pointed out the above, the usual discussion of such models is beyond the grasp of less sophisticated students and are therefore left out of most lectures. My model has the advantage that I can incorporate questions of entry deterrence and the Dixit-Spence model in a simple yet intuitive way.

I can also extend my discussion to different modes of entry into the product market. This is a topic that is reserved for graduate courses. Consider a game with two firms who apply for a patent and entry into the product market is dependent on when the firms get the patent. Depending on when the patent is granted entry into the product market could be simultaneous or sequential. See Aoki and Prusa (1997) and Chaudhuri (2000) for representative models. Normally a topic like this would be completely beyond the scope of an undergraduate lecture, but using my simple model the topic can be handled and explained to students without a lot of complicated algebra. Moreover my model can accommodate more than two firms and a variety of entry sequences into the product market – sequential and simultaneous as I show in Section 3.2.

Thus the simple model I develop to provide an alternative explanation of the Stackelberg model serves a large purpose in enabling me to tackle a number of topics, such as entry sequence and entry deterrence, which would normally be far beyond the scope of an undergraduate course. Moreover being very simple, the model has the added advantage of appealing to students who may not be mathematically sophisticated.

## Section 5: Conclusion

In this paper I have provided a simple algebraic framework for teaching the Stackelberg model where firms move sequentially instead of simultaneously. This framework provides a different way of thinking about the Stackelberg model. The framework can be extended to undertake a discussion of various modes of entry. Moreover the model has the added advantage that it takes topics which can be technically sophisticated and provides an intuitive and computationally simple way of dealing with them. This makes the topics much more appealing to students who may not have a strong background in calculus and hence the framework has the potential of improving undergraduate instruction in the area.

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